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Probabilities and Payoffs

The Practicalities and Psychology of Expected Value

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Introduction

Finding securities with gaps between price and value is the foundation of generating excess returns in active investment management.[1] The difference between price and value, commonly called "variant perception" or "edge," comes from having a substantiated view that diverges from what the market reflects.[2] In theory, the size of an investment within a portfolio maximizes the benefit of edge while considering risk.

This is all simple in principle but difficult in practice. One of the main challenges is discerning the gap between price and value. Price is the relatively easy part. Buying or selling securities incurs transaction costs, and the magnitude of those costs depends on factors such as the liquidity of the security.[3] But price is transparent and investors can estimate market impact.

Value is the hard part. This is because value is really "expected value," which represents a range of potential payoffs with associated probabilities. Investing is an inherently probabilistic activity. The concept of expected value raises lots of issues that we will explore.

One of the most challenging aspects of understanding expected value is that excess returns can be the product of high probability events with relatively low payoffs, or low probability events with relatively high payoffs.[4] In other words, how often you are right is not all that matters. What is vital is how much money you make when you are right versus how much you lose when you are wrong.

We have called this the "Babe Ruth effect."[5] Ruth is considered one of the greatest baseball players of all time and yet was the career leader in strikeouts, a measure of offensive failure, when he retired. At the same time, his slugging percentage, which assesses batting productivity, remains the highest in the history of Major League Baseball. What was good in his results more than offset what was bad. We will look at the frequency and magnitude of payoffs across asset classes.

Some markets have seen a shift in appetite from high probability, low payoff opportunities to low probability, high payoff ones. In betting on horse races, there has been a rise in "exotic" wagers, which can include several horses and multiple races, versus simple win, place, and show bets.[6] In sports betting, parlay bets, also wagers on multiple outcomes, have grown relative to simple point spread or over/under bets.[7] And there has been a surge in the trading of short-dated options in equity options markets.[8]

In this report, we discuss some of the issues with the calculation of expected value, what the payoff picture means for investing, the implications of volatility drag, the psychology of dealing with probabilities and payoffs, and how these ideas can be helpful for investing in various asset classes.

We focus on equities primarily but the thinking applies to credit and derivatives as well.

What to Expect from Expected Value

A calculation of expected value requires a quantification of potential payoffs and the probability of each payoff occurring. The sum of the probabilities must be 100 percent. The expected value is the sum of the product of each payoff and its associated probability (see exhibit 1 for a simplified example of the expected value of a drug).

Exhibit 1: Expected Value Calculation of a Hypothetical Drug

Scenario	Probability	Payoff	Weighted value	Expected value
Breakthrough	10%	$2,500,000	$250,000	
Above average	20%	$1,200,000	$240,000	
Average	40%	$137,500	$55,000	$550,000
Below average	20%	$20,000	$4,000	
Dog	10%	$10,000	$1,000	
	100%			

Source: Counterpoint Global based on David Kellogg and John M. Charnes, "Real Options Valuation for a Biotechnology Company," Financial Analysts Journal*, Vol. 56, No. 3, May/June 2000, 76-84.*

Expected value calculations span from the simple to the very complex. As Warren Buffett, chairman and chief executive officer of Berkshire Hathaway, has said, "Take the probability of loss times the amount of possible loss from the probability of gain times the amount of possible gain. That is what we're trying to do. It's imperfect, but that's what it's all about."[9]

Economists typically translate expected value into expected utility, an idea that Daniel Bernoulli, a mathematician, introduced in 1738. Utility is a measure of satisfaction and varies from person to person based on individual preferences. Most people exhibit risk aversion, meaning that the marginal utility of wealth diminishes as wealth increases.[10] Expected value is a key concept in decision-making under uncertainty, but economists recognize that individuals make choices based on different utility functions, which lead to a range of preferences.

Most teachers start their lessons about expected value using examples with set probabilities and payoffs. For instance, the expected value of the toss of a fair coin that pays $2 for heads and $1 for tails is $1.50 ([0.50 × $2] + [0.50 × $1] = $1.50). But investing is vastly more complex than the toss of a coin, the roll of a die, or the turn of a playing card. The mindset carries over but the math does not. Overapplying these simple cases to the more complicated ones is called the "ludic fallacy"—ludus is Latin for game.[11]

Frank Knight, an economist, made this point by distinguishing between “risk” and “uncertainty.” With risk, according to Knight, “the distribution of the outcome in a group of instances is known.” This is not true with uncertainty “because the situation dealt with is in a high degree unique.”[12] Risk includes the notion of harm whereas uncertainty need not reflect loss. Most of what investors deal with is Knightian uncertainty, although it is best to think of the ability to set probabilities and payoffs along a continuum from the obvious to the impossible.

During a press briefing in 2002, Donald Rumsfeld, then U.S. Secretary of Defense, answered a question by distinguishing between “known knowns” (“things we know we know”), “known unknowns” (“we know there are things we do not know”), and “unknown unknowns” (“the ones we don’t know we don’t know”). He added that the unknown unknowns is the “category that tends to be the difficult one.”[13]

Richard Zeckhauser, an economist and champion bridge player, writes, “The essence of effective investment is to select assets that will fare well when future states of the world become known.”[14] He notes that the efficient market hypothesis posits that probabilities and payoffs are established and, as a result, smart investing is an exercise in optimization.

The key to financial success when dealing with unknowns and ignorance, a good description of most investing, is the ability to assess probabilities and payoffs. Decision theory becomes more important than optimization.

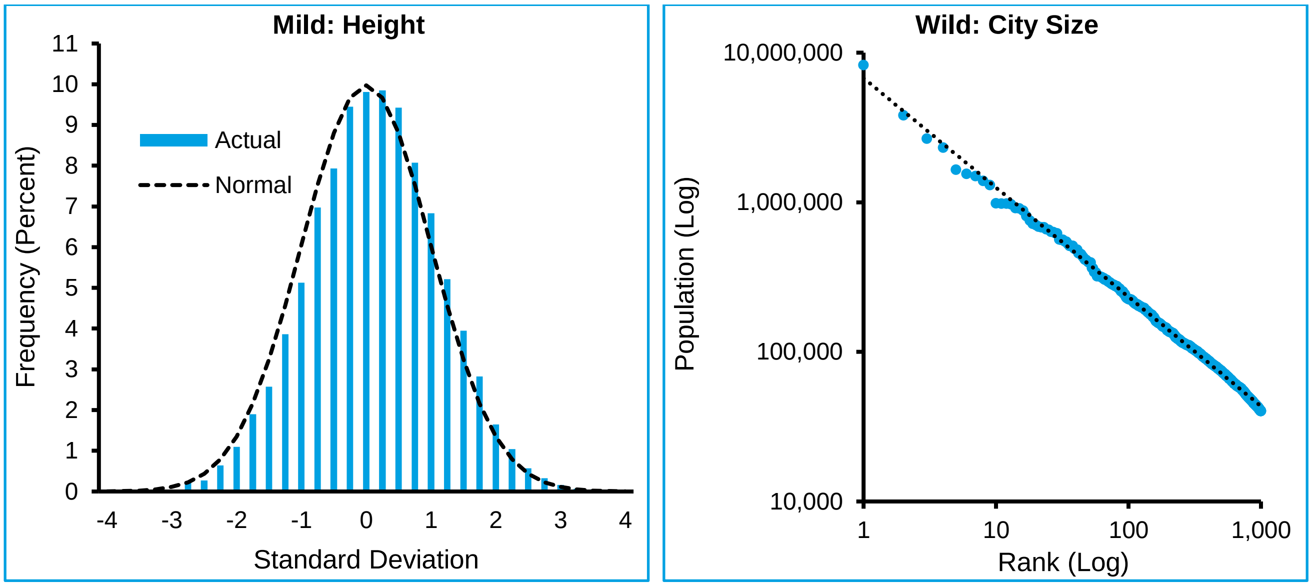
With these thoughts as background, we will take a closer look at payoffs and probabilities, the determinants of expected value.

Payoffs. Payoffs reflect the future states of the world and can range from the very simple to the highly complex. A common mistake in decision-making is “overprecision,” a form of overconfidence that occurs when someone is too confident in their views and therefore fails to consider a sufficiently wide range of alternatives.[15] Here are some points to keep in mind when assessing the likelihood that particular future states of the world will come to pass:

- **Consider the shape of the distribution of payoffs.** Benoit Mandelbrot, a renowned mathematician, used the terms “mild” and “wild” to distinguish between the ranges of future states.[16] Mild states can generally be captured with a normal, bell-shaped distribution. The distribution of the height of people, for instance, is mild, with the ratio between the tallest and shortest humans on record being five-to-one (left panel of exhibit 2). Statistical concepts such as mean (average) and standard deviation are useful in expressing mild states.

  Wild states are often power laws, where few very large outcomes have a disproportionate impact on the distribution.[17] Examples include the distribution of wealth and city size (right panel of exhibit 2). As a case in point, the ratio between the population of the largest city (New York, New York) and the one-thousandth (Dekalb City, Illinois) in the U.S. is 205-to-1. Mean and standard deviation are not useful in expressing the outcomes of these systems.

Exhibit 2: Mild and Wild States: Human Height and City Size



Source: Statistics Online Computational Resource Human Weight/Height Dataset and U.S. Census Bureau.
Note: Height of 25,000 people; cities in the United States.

Nassim Taleb, an author, popularized the idea of a "black swan," which he defines as an event that is an outlier, is consequential, and that humans try to explain after the fact.[18] Black swans are from the domain of unknown unknowns.

Many outcomes that investors call black swans are really what he calls gray swans, or known unknowns. For example, a large and devastating earthquake would be an outlier and consequential. But geologists have a good sense of the distribution of earthquake magnitudes even if they do not know exactly when or where an earthquake will occur.

The Stoics, ancient philosophers who believed in a life led well, advocated "premeditatio malorum"—the pre-meditation of evils. Seneca, a Stoic, wrote the following about adverse events that are unanticipated: "The fact that it was unforeseen has never failed to intensify a person's grief. This is a reason for ensuring that nothing ever takes us by surprise."[19] The point is to prepare for all eventualities.

- **Be mindful of the "grand ah-whoom."**[20] Phase transitions, where small changes in a cause lead to large effects, are pervasive in complex systems such as businesses and markets. Think of cooling water that starts at a temperature just above freezing. As the temperature drops below the point of freezing—ah-whoom—the liquid turns into a solid. A modest change has a large impact.

  Jay Forrester, a professor at Massachusetts Institute of Technology who taught system dynamics, developed the beer game to illustrate how small decisions can amplify into big effects. There are four teams (manufacturer, distributor, supplier, and retailer) and assumed lags between when the orders are received and when the beer is delivered. The goal is to meet consumer demand while minimizing back orders and inventory.

  Bullwhip effects, where relatively small distortions in demand create inefficiency throughout the supply chain, commonly emerge from playing the game. Bullwhip effects occurred in multiple supply chains during and following the COVID-19 pandemic. One widely discussed case was toilet paper: an initial spike in consumer purchases caused retailers to order much more product, which signaled high demand to manufacturers who then ramped production. Demand then normalized, which led to oversupply at warehouses, distribution centers, and stores. That, in turn, led to lower retailer orders.

Markets also have phase transitions when there is a diversity breakdown. Investors with diverse approaches interacting with one another generally produce accurate asset prices, as the wisdom of crowds predicts.[21] But from time to time, diversity breaks down and the beliefs of investors align, resulting in booms or busts.

The essential insight is that the trend in the asset price remains in place even as diversity declines and fragility rises. It is only at a critical point that there is a strong reversal—the bubble pops—and diversity is restored. Here again, a small change in the state of the system leads to a large effect on the system.

- **Control and reversibility.** Because payoffs reflect future states of the world, it is important to understand when the payoffs are expected to happen (time horizon), whether the decision-maker can alter the payoffs (control), and if the investment can be exited at an acceptable cost (reversibility).[22] Reversibility is closely tied to liquidity, the cost of turning cash into an asset or an asset into cash. That cost is low in liquid markets and high in illiquid markets.

  To illustrate, consider the differences between an investment made by a company and one made by an equity investor. Investments by companies such as building a data center or acquiring another company tend to be long-term because the cost of reversal is high. Offsetting that illiquidity is some control over the potential payoffs. Companies can act if the potential payoffs appear to be following an unsatisfactory path, including tweaking a product offering, changing the pricing, refining the marketing strategy, or replacing the managers in charge of the business.

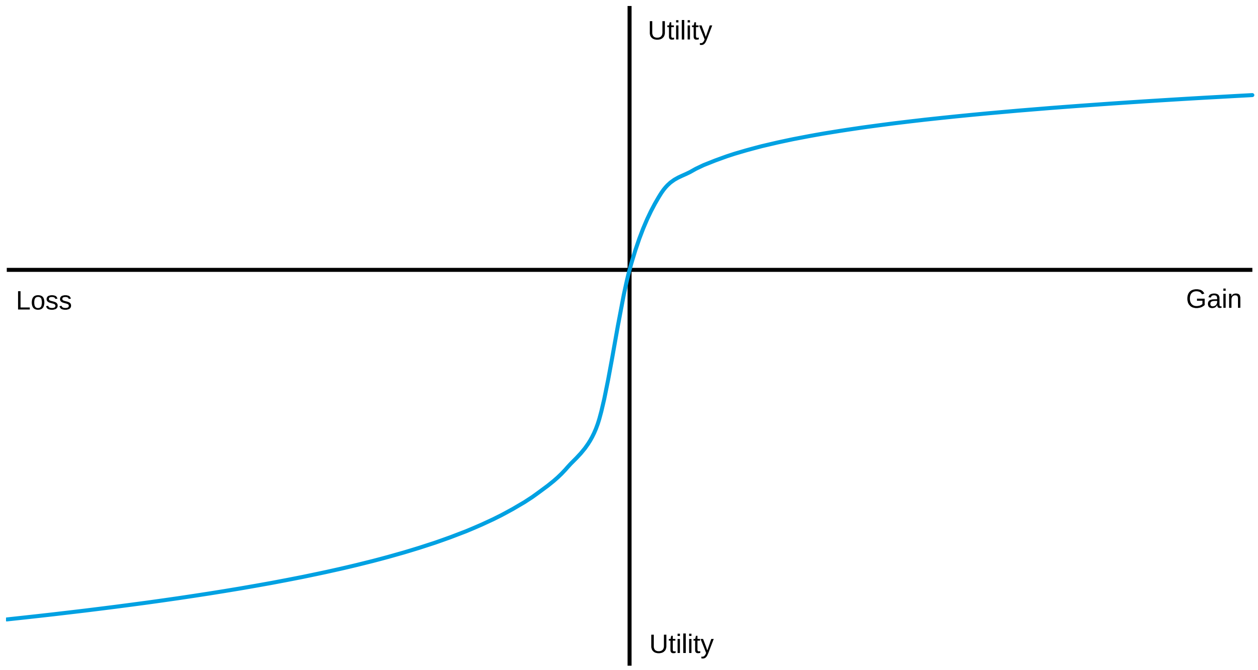
  Public equity investors have much more liquidity but commonly have limited control over payoffs. Note that even activist investors, who seek to improve the payoffs by promoting change at the companies they invest in, often need to have a sizeable stake in the company to establish credibility. More control requires less reversibility.

- **Asymmetric payoffs.** Fundamental investors commonly seek opportunities where the magnitude of payoffs on the downside are smaller than those on the upside. In other words, there are more potential gains than losses (naturally, the probability of the payoffs is also crucial). In some cases, certain measures of valuation may suggest a limit to downside payoffs. These include cash balances, tangible book value, and free cash flow yield.

  While our focus is on equities, the upside payoffs are generally capped for bonds. The upside payoff for a straight bond held to maturity is the present value of coupon payments plus the return of principal. For this reason, equity investors tend to focus on upside payoffs and bond investors are inclined to dwell on the avoidance of loss. This is why Benjamin Graham and David Dodd, authors of *Security Analysis*, called bond selection "primarily a negative art."[23]

  Daniel Kahneman and Amos Tversky, professors of psychology, developed "prospect theory" in part as an effort to explain the observation that people suffer more from losses than they enjoy gains of comparable size, leading to "loss aversion."[24] Relative to a reference point, people tend to be risk-averse in the realm of gains and risk-seeking in the realm of losses (see exhibit 3). Empirical data back the point that prospect theory explains investor behavior better than classic expected utility theory does.[25]

Exhibit 3: Kinked Utility Function As Described By Prospect Theory



Source: Counterpoint Global based on Daniel Kahneman and Amos Tversky, "Prospect Theory: An Analysis of Decision under Risk," Econometrica*, Vol. 47, No. 2, March 1979, 263-292.*
Note: Assumes utility of wealth equals the log of wealth, U(w) = ln(w), and a loss aversion coefficient of 2.0.

There are categories of investments where the payoffs have a low probability of a large positive or negative outcome. Specifically, investors can buy investments with characteristics similar to a lottery (lose a little and potentially make a lot) or sell those similar to insurance (make a little and potentially lose a lot).

Research shows that investors commonly overprice stocks with lottery characteristics because they overweight the probability of a high payoff.[26] Some financial economists have concluded that it is better to sell, rather than buy, investments with lottery- and insurance-type payoffs.[27]

This thinking can be expanded from individual opportunities to investment strategies. Selling investments with lottery or insurance payoffs means making a little money most days and losing lots of money from time to time (blowup). Buying investments with lottery payoffs means losing a little money most days and making lots of money every now and then (bleed).

Nassim Taleb believes in the bleed strategy and argues that extreme outcomes are underpriced. But he concedes that a financial firm may prefer to make money steadily even at the risk of a blowup.[28] An example is Long-Term Capital Management, a hedge fund, which had returns well in excess of the market from 1993 to early 1998 but then plummeted.

- **Internal versus external factors.** When considering payoffs and probabilities for the stock of a company, investors commonly and appropriately focus primarily on the drivers of value for that firm. For example, an analyst may consider different scenarios for measures of corporate performance such as sales growth and operating profit margins and estimate the payoff per share for each scenario.

  One of the most important findings in finance is that changes in stock market prices are greater than what is justified by changes in fundamentals.[29] A pair of academic papers looked at the largest moves in the

S&P 500, an index of about 500 of the largest stocks in the U.S., from 1941 to 2012 and then examined the explanations offered by the business press after the fact.[30]

They found that external shocks, generally related to international relations or political developments, could explain some of the moves. Call these exogenous risks. But more strikingly, a large percentage of the big moves did not seem to have a corresponding causal event but rather seemed to have come from within the system. The authors of one of the studies wrote, "On most of the sizable return days, however, the information that the press cites as the cause of the market move is not particularly important. Press reports on subsequent days also fail to reveal any convincing accounts of why future profits or discount rates might have changed."[31] Call these internal, or endogenous, risks.[32]

As we saw with ah-whoom moments, large scale changes can occur from within the system without an obvious external cause. This empirical reality is worth considering when assessing payoffs.

Probabilities. Philosophers, statisticians, and mathematicians have debated the meaning of probability for centuries. Some have argued that probability is a subjective assessment that fails to reflect a real quantity, and hence does not really exist.[33] That said, it is useful to consider probabilities in evaluating opportunities. Here are some points to keep in mind when assessing the likelihood that particular future states of the world will come to pass:

- **Methods to set probabilities.** There are broadly three approaches to setting probabilities: frequentist, propensity, and degrees of belief (subjective).[34] These camps do not always see eye to eye.[35]

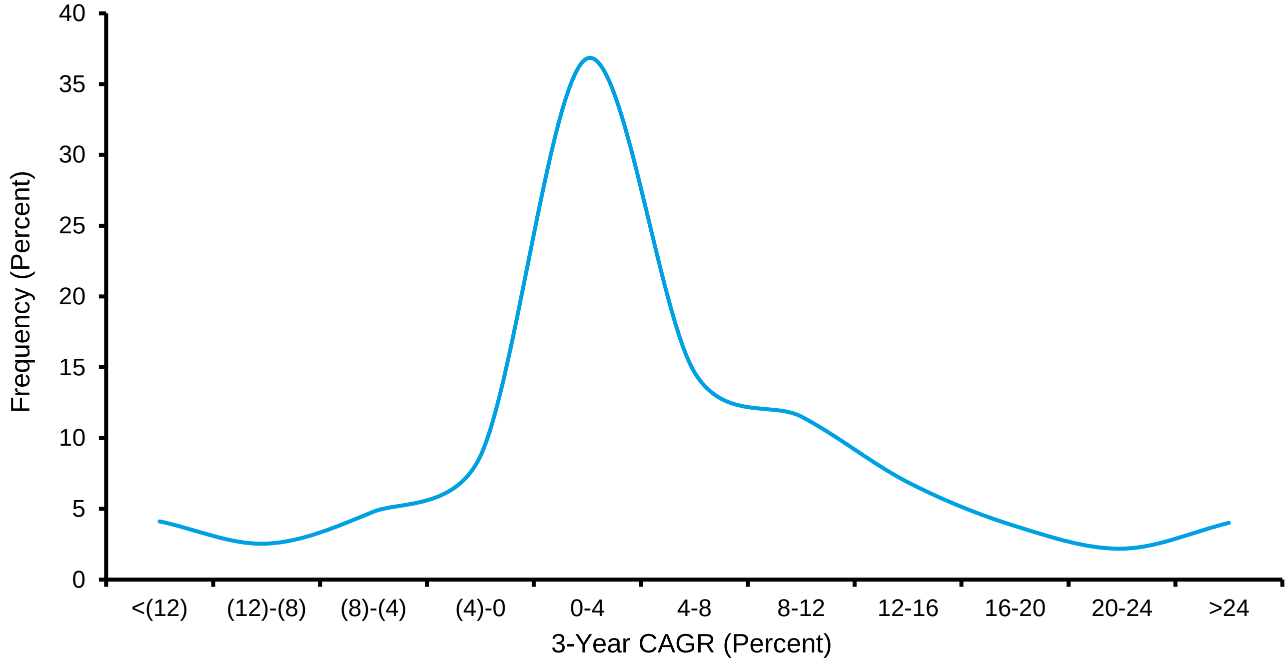
  The frequentist sets probabilities based on a large sample of outcomes for a particular reference class. The likelihood of a six appearing with the roll of a die is one-in-six based on a huge number of observations of die rolls.

  The propensity approach judges probability based on the properties of the object under consideration. The probability of rolling a six is 16.7 percent, reflecting the physical nature of a die as a perfect cube.

  Degrees of belief measures the subjective probability an individual assigns to an outcome. This probability can be quantified through an analyst's willingness to bet.[36] An analyst who believes the likelihood of rolling a 6 is 16.7 percent and is neutral to risk would be indifferent between doing nothing and betting $1 on rolling a 6 if the payoff was $6 ($1 = .167 × $6). Investment analysts deal mostly with subjective probabilities. An initial degree of belief is called a "prior." The prior that a die will show a six is based on a person's assessment before any new information is revealed.

  One sensible approach for investors is to use base rates as a way to inform prior probabilities and then update those probabilities as additional information becomes available. A base rate reflects the probabilities and payoffs for a specific reference class. To illustrate, exhibit 4 shows the distribution of three-year compound annual growth rates of sales for U.S. companies over the past 63 years.

Exhibit 4: Three-Year Compound Annual Sales Growth Rates for U.S. Companies, 1962-2024



Source: FactSet, Compustat, and Counterpoint Global.
Note: Companies listed on the New York Stock Exchange, NASDAQ, and NYSE American stock exchanges, with a minimum $1 million of sales in 1962 dollars; nominal growth; CAGR=compound annual growth rate.

One essential point is that probabilities and payoffs are dynamic. That means that new information will justify a revision in prior probabilities. The formal way to do this is with Bayes' Theorem, which tells you the probability that a prior belief is true conditional on some event happening. While the math is useful, what is more important is an openness to updating your views.[37]

Research suggests that confirmation bias, the tendency to dismiss, discount, or disavow new information in favor of a prior view, can impede proper updating.[38] It helps to think like a "fox"—one who knows a little about a lot—rather than a "hedgehog"—one who knows one big thing. Foxes update their views more readily than do hedgehogs, who prefer to fit the facts to their worldview.[39]

Investors deal mostly with subjective probabilities. These are useful if set carefully and revised appropriately. But there is an additional layer of nuance: confidence in probability.

- **Confidence in probability.** Probability and confidence are distinct concepts that often get combined, unwittingly, in investment analysis. You can think of probability as an estimate of the chances of a payoff and confidence as "the degree to which an analyst believes that he or she possesses a sound basis for assessing uncertainty."[40] Psychologists call the probability assigned to a payoff "first order uncertainty." A reasonable range of probabilities for first order uncertainty is called "second order uncertainty." It reflects uncertainty about an uncertain payoff.

  Jeffrey Friedman, a professor of government, and Richard Zeckhauser describe three dimensions to confidence: reliability of available evidence, range of reasonable opinion, and responsiveness to new information.

  Reliability of available evidence answers the question, "Can I defend this estimate with a substantial amount of information?" A large amount of relevant knowledge provides a sound basis for assessing risk and uncertainty. Fact and opinion are both important in investing. This dimension focuses on fact, which should carry more weight than opinion in assessing confidence.

Range of reasonable opinion addresses the query, "Might reasonable people give substantially different answers to this question?" This comes into play whenever an analyst is considering probabilities and payoffs that relate to a complex adaptive system, where inputs and outputs are not linked linearly.

Complex adaptive systems describe a network of adaptive agents that interact with one another creating a system that is emergent. In these cases, analysis of the underlying agents does not predict outcomes.[41] Prominent examples include climate systems, stock markets, and the economy. This is why the accuracy of forecasts in these domains tends to be poor.

Responsiveness to new information reflects on the question, "Is my view likely to change substantially if I study the subject further?" The answer is based on how firmly an analyst holds a prior view and whether the benefit of new information is worth the cost and time to access it. This introduces the constraints of resources and time. Responsiveness to new information compels the decision-maker to think about the cost and benefit of pursuing additional information.

Confidence in probabilities can be important when making investment decisions. For instance, two opportunities may have the same discount to expected value, but the confidence in the probabilities for one may exceed that of the other. That insight may be important for position sizing within a portfolio or for risk assessment.

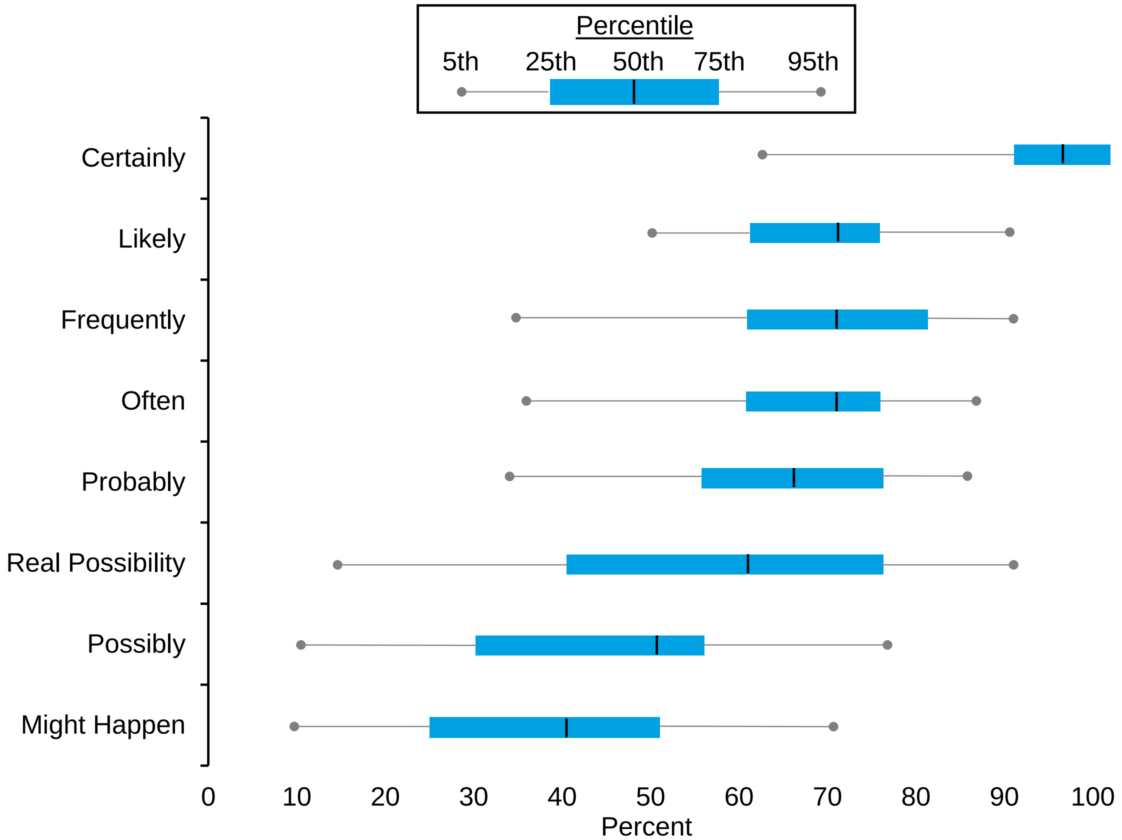
Words to probability. A perceived variant perception motivates most investment decisions based on fundamentals. The challenge is that the articulation of the variant perception relies too often on vague words rather than numerical probabilities. Phrases such as "we believe," "the chance is good," and "there is a real possibility" are examples of this type of communication. A better approach is to quantify the variant perception.[42]

There are at least two problems with using words instead of probabilities. The first is that people assign different probabilities to the same word or phrase. This introduces the potential for miscommunication. Exhibit 5 shows some of the results of a survey of more than 3,000 respondents who were presented with words or phrases, in random order, and asked to assign probabilities to each.

While some words translate into consistent probabilities, the variation is huge in other cases. For example, the term "might happen" evoked a range between 10 and 70 percent (setting aside the lowest and highest 5 percent of the responses). The cognate words and phrases of "possible" are particularly nettlesome as they are interpreted to express a wide range of probabilities.[43]

The second problem is that using words can allow an investor to skirt accountability when he or she is wrong. The ambiguity in words provides an investor the opportunity to craft a narrative that explains the wrong judgment. Examples include the close call ("I was almost right"), bad timing ("my prediction will be right but the timing was off"), and the unexpected ("an unforeseen event messed up my forecast"). We tell stories to ourselves and others to paper over our poor predictions. Barbara Mellers, a professor of psychology at the University of Pennsylvania, says, "We find prediction really hard, but we find explanation fairly easy."[44]

Exhibit 5: How Words or Phrases Are Interpreted as Probabilities



Source: Counterpoint Global and www.probabilitysurvey.com.

- **Feedback and calibration.** Skill acquisition requires timely and accurate feedback. You need to know where and how you were wrong to improve on the next try. The challenge with investing and business is that the feedback can be noisy and come with a lag. This impedes learning.

  In decision-making, calibration measures the degree to which someone's subjective assessment, a measure of confidence, aligns with how often they are correct. Exhibit 6 shows a classic example: thousands of participants answered 50 true-false questions and indicated their confidence in their response for each. The exhibit shows that the confidence of the participants exceeds their correctness in the aggregate. For example, when they are 100 percent sure they know the answer, participants are correct only 77 percent of the time. Psychologists have replicated this finding many times.

  Note that being well-calibrated does not mean knowing the answer to each question. It is about being as close as possible to the 45-degree angle line between confidence and being correct. Excellent calibration comes from knowing what you know and knowing what you do not know.[45]

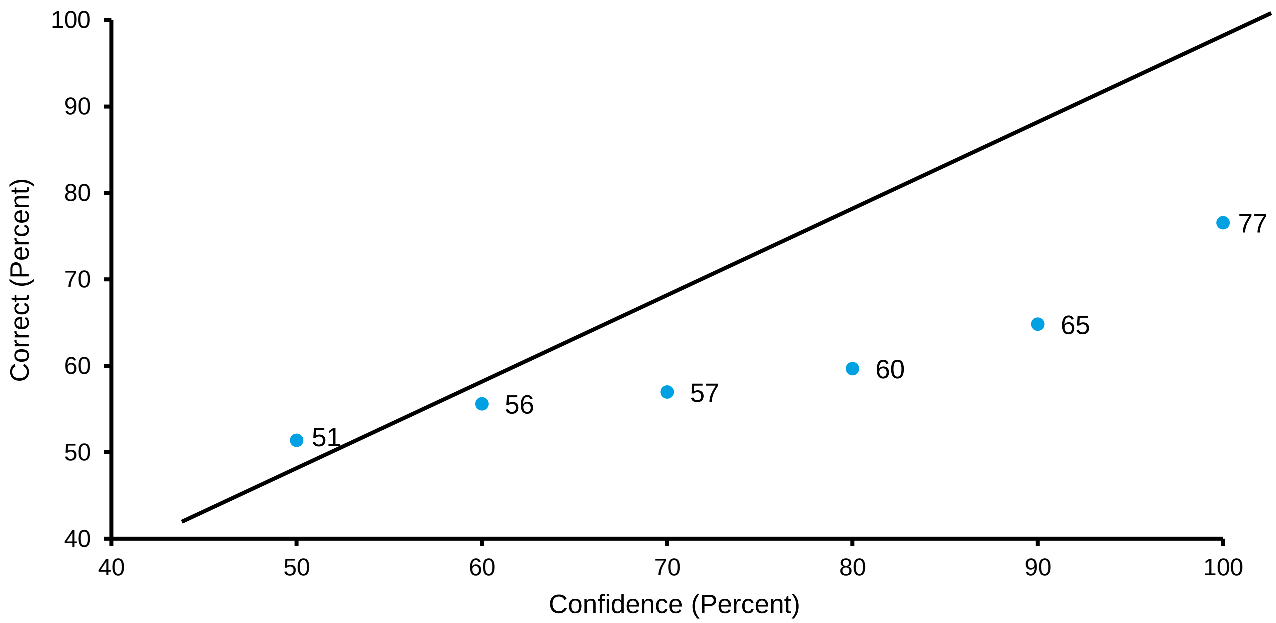
  The question is whether feedback helps improve calibration. The answer is yes.[46] In one study, researchers asked forecasters to make predictions about meteorological data, such as wind speed, visibility, and precipitation, and measured their calibration. Consistent with the results in exhibit 6, the confidence in the forecasts exceeded the correctness. But after receiving extensive feedback, the forecasters improved their calibration the next year, with their results falling closer to the diagonal line.[47]

We noted that feedback in investing and business is impeded by noise and lag time between forecast and outcome. The way to deal with noise is to keep score using probabilities instead of words. The way to deal with the lag time is to break down a thesis into subcomponents that are relevant over shorter time horizons.

A variant perception, or investment thesis, can almost always be distilled into outcomes that are objective, within a specific time horizon, and occur with an estimated probability. ("The company will divest division X for $1 billion or more by the end of the year with an 80 percent probability.") These three ingredients allow you to score the quality of a forecast. The appendix discusses the Brier score, a common way to measure forecasting accuracy.

We have found that asking investors to assign probabilities to payoffs with the intention of keeping score prompts useful introspection. Documenting decisions also allows for an audit of the investment process. Some investments do well even when the thesis is wrong (bad process, good outcome) and others do poorly when the thesis is solid (good process, bad outcome). Feedback and calibration help improve the process, which is the best way to increase the chances of satisfactory outcomes over time.[48]

Exhibit 6: Participants Are Overconfident on Average in Probability Assessments



Source: www.confidence.success-equation.com.

Best practices. Now that we have discussed considerations surrounding the setting of payoffs and probabilities, we touch on some best practices for translating these ideas into action.

- **Use base rates.** When modeling expected corporate results, investors commonly gather lots of information (e.g., financial filings, communication with management, sell-side research, company financial guidance, surveys, expert calls), which they combine with their own experience and judgment, and project into the future.[49] This practice introduces a number of potential biases, including confirmation, overconfidence, recency, and availability.[50]

  Integrating base rates overcomes some of the limitations of this approach. Rather than considering each problem as unique, the base rate considers the results of a relevant reference class. Instead of asking, "what do I think will happen?" the base rate approach asks, "what happened when others were in this situation before?" Psychologists have shown that decision-makers often neglect base rates and that combining a subjective assessment and the base rate improves the quality of forecasts.[51]

One hurdle in thinking this way is that we are much more drawn to stories than we are to statistics.[52] Experiments show that the impact on beliefs fades much slower for stories than it does for statistics.[53] We are more likely to remember and believe a story than a statistic.

The main challenge in applying base rates effectively is identifying an appropriate reference class. There are specific steps in the process, although it remains a combination of science and art.[54] Investors commonly neglect base rates because they are not readily available. But financial results for companies is one area where it is practical to use base rates.

Forecasts of sales growth, generally the most important driver of shareholder value, are a good illustration, as we saw in exhibit 4. The distribution of sales growth rates for a population of companies is reasonably stable. As a result, there is value in considering sales growth expectations relative to past experience, and academics have developed approaches to creating robust reference classes.[55]

Base rates are beneficial, but it is important to keep in mind that they are often dynamic distributions rather than fixed facts. Warren Buffett, in his 2001 letter to shareholders, distinguishes between experience and exposure.[56] Experience is reflected in base rates while exposure considers the possibility of something that has never happened before. Buffett's comment was in the context of the insurance industry prior to the terrorist attacks in the U.S. in 2001. The industry had no experience in attacks on this scale but did have exposure.

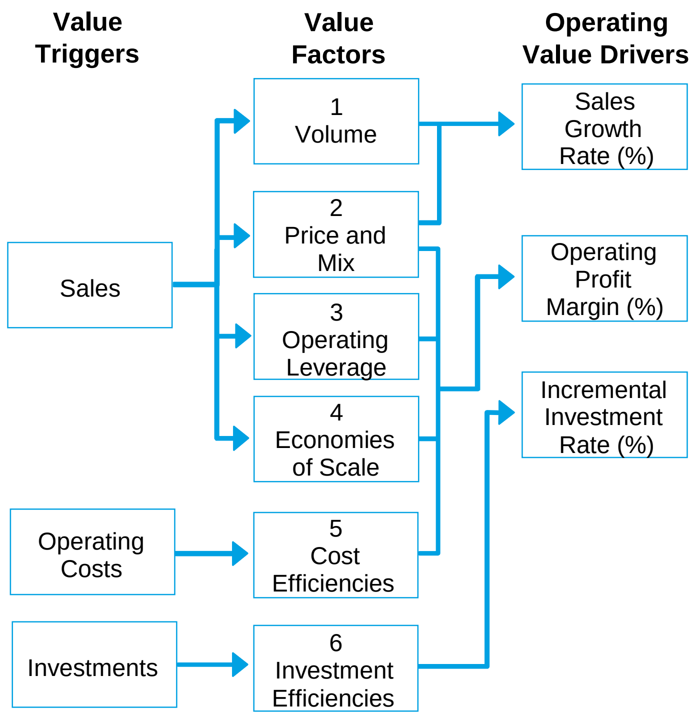
This is important for investors. Understanding potential adverse payoffs is essential. But appreciating potential upside payoffs is relevant as well. Indeed, our research suggests that the rise of investments in internally-generated intangible assets has fattened the tails of the distribution of sales growth: some companies are growing faster, and others shrinking faster, than companies have in the past.[57]

- **Sensitivity and simulation.** Investors and businesses often create payoffs and probabilities in the form of basic "bull," "bear," and "base" cases. Consistent with overprecision, the range of payoffs is often too narrow and the probabilities are set simplistically.[58] Further, analysts produce sensitivity analysis that fails to capture the essential interactions between the drivers of business performance.

  Our recommendation is to use the expectations infrastructure (exhibit 7), which creates a mapping of the interactions between the value triggers—sales, operating costs, and investments—and the ultimate operating value drivers. The crucial point is that the elasticity of operating profit to changes in sales differs a great deal by industry and company. As a result, analyst earnings forecasts can be very inaccurate, especially in the case of declining sales.[59]

  A further suggestion is to use more than three scenarios. Additional complexity does have a cost but we would argue that the insight gleaned from considering, say, five scenarios is a worthwhile trade-off. The main benefit is offsetting the risk of overprecision. Thoughtful use of Monte Carlo methods, a form of simulation that produces payoffs based on draws from a distribution, can also lead to a deeper appreciation of potential probabilities and payoffs.[60]

Exhibit 7: The Expectations Infrastructure



*Source: Michael J. Mauboussin and Alfred Rappaport, Exp*ectations Investing: Reading Stock Prices for Better Returns—Revised and Updated *(New York: Columbia Business School Publishing, 2021), 46.*

- **Margin of safety.** Ben Graham, the father of security analysis, suggested that the secret of sound investment could be distilled into three words, "MARGIN OF SAFETY" (capitalization original).[61] Margin of safety is the difference between value and price, and the point is that you want to have a sufficient gap to improve the odds of generating excess returns as that gap narrows as well as to compensate for "miscalculations" in analysis or "worse than average luck."[62]

  Expected value is the best way to think about value. Graham allowed that even investments with an attractive margin of safety only improve the chances of a profit but do not eliminate the possibility of a loss. For this reason, Graham suggested that portfolio diversification is the "companion" to the principle of margin of safety, reckoning that the more investments that have attractive gaps between value and price the more likely that the overall portfolio will fare well.

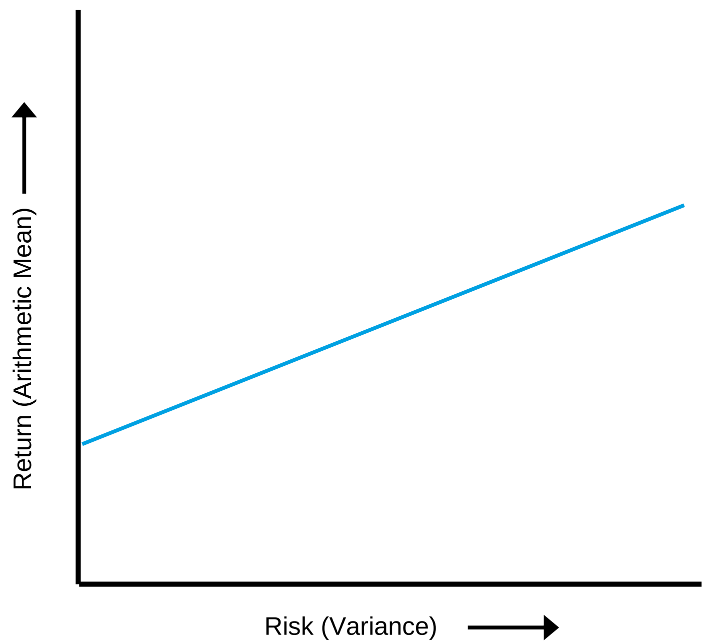
Now that we have discussed some of the issues surrounding how to think about, and calculate, the inputs to expected value we turn our attention to how these ideas apply under different considerations.

Expected Value and Decisions

The process of estimating expected value through payoffs and probabilities provides a way to quantify variant perception, or edge, and compels useful thought and analysis. The next question is how to translate that work into action.

Harry Markowitz, an economist and recipient of the Nobel Prize in Economics, came up with one answer to this based on mean/variance optimization.[63] The idea, which fits with intuition and experience, is that risk and reward are related in a linear fashion. The security market line (exhibit 8) shows this visually. The return is the mean, or average, arithmetic return from an asset or portfolio. The risk is variance, a measure of how far points on the distribution are spread from the average.

Exhibit 8: The Security Market Line



Source: Counterpoint Global.

Markowitz's basic point is that an investor who cares about risk will seek the highest return for a given level of risk or the lowest risk for a particular level of return. For example, if two portfolios have the same return but one has lower risk than the other, the investor will select the portfolio with the lower risk. Markowitz showed that the portfolios with the best risk and reward characteristics fall along the "efficient frontier."

In theory there is no universally optimal portfolio because investors differ in their preferences. But portfolios that fall at a distance from the efficient frontier are suboptimal. Mean/variance optimization is useful because you can find an appropriate portfolio if you know your appetite for risk. Modern portfolio theory holds that the "market portfolio," the market-weighted value of all investable assets, is mean/variance optimal.

An essential point is that mean/variance optimization generally assumes you are deciding based on one period. But the approach is different if you consider multiple time periods and your goal is to maximize the likelihood that you will have the most money at a date far in the future.

This insight came from John Kelly, a physicist who used information theory to develop a strategy for optimal betting over the long term.[64] Kelly noted that if a gambler made one bet of one dollar per week but could not reinvest his winnings, he should maximize expected value. This is Markowitz.

But the math changes if the winnings are reinvested from one period to the next. Instead of seeking the outcome with the best arithmetic mean, the objective is to find the opportunity with the highest geometric mean. This is called the Kelly criterion, or Kelly strategy. In this case, subjective preference does not determine risk. Rather, there is a knowable amount of risk that provides the best results in the long run.

The arithmetic mean is the sum of the values divided by the number of values. For example, the annual arithmetic mean return for the S&P 500 was 11.9 percent for the 20 years ended in 2024.

The geometric mean return represents the average rate of return per period, accounting for compounding. For the 20 years ending in 2024, the annual geometric mean return for the S&P 500 was 10.4 percent.

The difference between the arithmetic mean, a simple average, and the geometric mean arises from the variance, or volatility, in returns. If there is no volatility in the returns, the arithmetic and geometric means are equal. However, as volatility increases, the arithmetic mean will always be higher than the geometric mean because of the compounding effect of positive and negative returns.

An illustration can help demonstrate the difference between mean/variance optimization and geometric mean maximization. We draw this example from *Fortune's Formula*, a wonderful book by William Poundstone that tells the story of Kelly's research and its implications.

Exhibit 9 shows the probabilities and payoffs for three investment opportunities. Poundstone suggests thinking of them as wheels of fortune, each with six outcomes, that you spin to determine your outcome.

Exhibit 9: Probability and Payoffs for Three Opportunities

	A		B		C	
	Probability	Payoff	Probability	Payoff	Probability	Payoff
	50%	$1.00	50%	$2.00	50%	$3.00
	50%	$2.00	17%	$0.00	50%	$0.50
			17%	$1.00		
			17%	$3.00		
Arithmetic mean		$1.50		$1.67		$1.75
Variance		$0.30		$1.07		$1.88
Geometric mean		$1.41		$0.00		$1.22

Source: William Poundstone, Fortune's Formula: The Untold Story of the Scientific Betting System That Beat the Casinos and Wall Street *(New York: Hill and Wang, 2005), 198.*

The probabilities and payoffs allow us to calculate the arithmetic and geometric means. We can see that of the three opportunities, the expected value, or arithmetic mean, is lowest for A, in the middle for B, and the highest for C. If you bet the same amount every time you should maximize expected value. Markowitz would say that the best choice is a function of an individual's preference. But opportunity C is the most attractive, all else being equal.

If your profits or losses in the prior period are reinvested into your bankroll, you should use the Kelly criterion and maximize the geometric mean. In this case, A is the most attractive opportunity and C is second best. To make this conclusion vivid, Poundstone calculates that a person who starts with $1, bets weekly for a year, and reinvests profits, would see the bankroll grow to roughly $67 million with opportunity A (the geometric mean multiplier, G, is a measure of the rate of growth of a variable over multiple periods, and equals $\sqrt{1 \times 2}$ = 1.41, and $\$1.41^{52}$ = $67,108,864) and to about $38,000 with opportunity C (G = $\sqrt{3 \times 0.5}$ = 1.22, and $\$1.22^{52}$ = $37,877). These results are not assured because of normal variance. But A will beat C every time given enough trials.[65]

Opportunity B has a positive expected value but a geometric mean of zero. This is the financial version of Russian roulette.[66] You will lose all of your bankroll with this strategy given a sufficient number of trials because one of the payoffs is nil. The lesson is that some strategies with positive expected value can still result in financial disaster, especially since real probabilities and payoffs are more opaque than those in this illustration.[67]

One way to think about this is that mean/variance optimization (Markowitz) focuses on diversification at a point in time and geometric mean maximization (Kelly) considers diversification over time.[68] Markowitz was fully aware of Kelly and related research and wrote favorably about it.[69]

Ergodicity economics, a field led by Ole Peters, a physicist, provides another way to think about this issue.[70] A process is ergodic if the ensemble average and the time average are the same. For instance, imagine 100 people flipping a fair coin simultaneously and recording the outcomes (ensemble). Now imagine flipping a fair coin 100 times in a row (time average). This process is ergodic because the expected outcomes are the same.

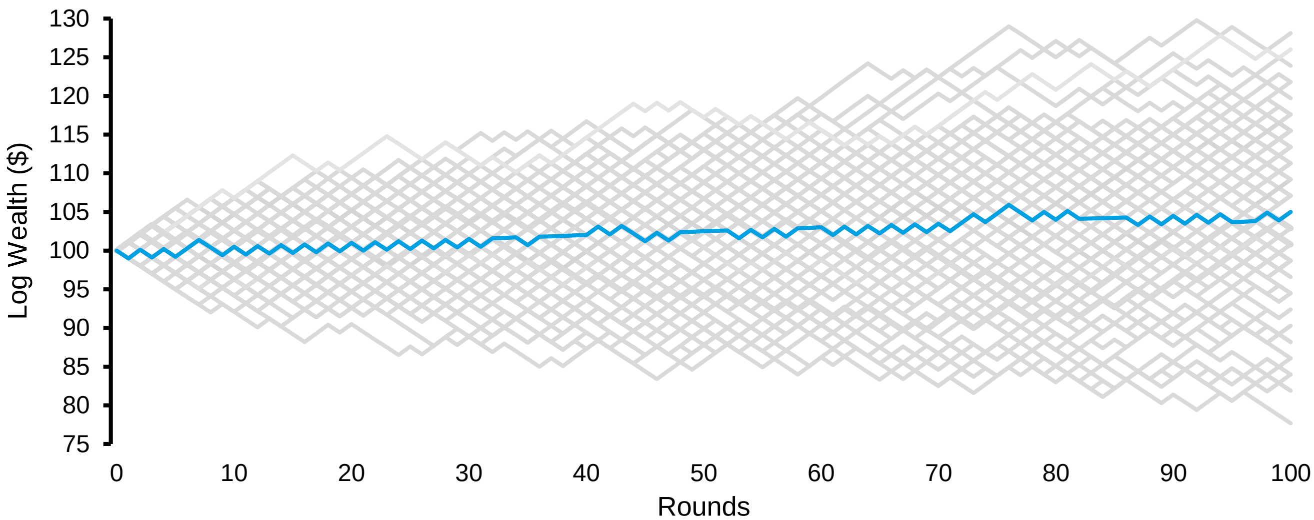
Consider that you pay $1 for the flip of a fair coin that pays $1.10 when it comes up heads and costs $1 when it comes up tails. This game has a positive expected value of $0.05 per dollar played ([0.50 × $1.10] + [0.50 × -$1.00] = $0.05), which leads to expected wealth of $1.05 [$1.00 + $0.05 = $1.05].

Assume that 100 people, each with $1, play simultaneously. Half will end up with $2.10 and the other half zero. This game has a positive expected value of $5.00 in the aggregate [100 x $0.05 = $5.00], and the expected wealth for the group is $105 ([50 × $2.10] + [50 × 0] = $105).

Now you alone start with $100 and play the game 100 times in a row. You will also see results split roughly evenly between heads and tails, and your expected wealth is the same at $105.

Exhibit 10 shows 100 runs of this game along with the median outcome (the average is virtually identical). This game is ergodic because the payoffs are arithmetic. The expected outcomes of the ensemble and time averages are the same and continue to converge as the number of rounds increase.

Exhibit 10: Median Wealth Change in an Ergodic Game



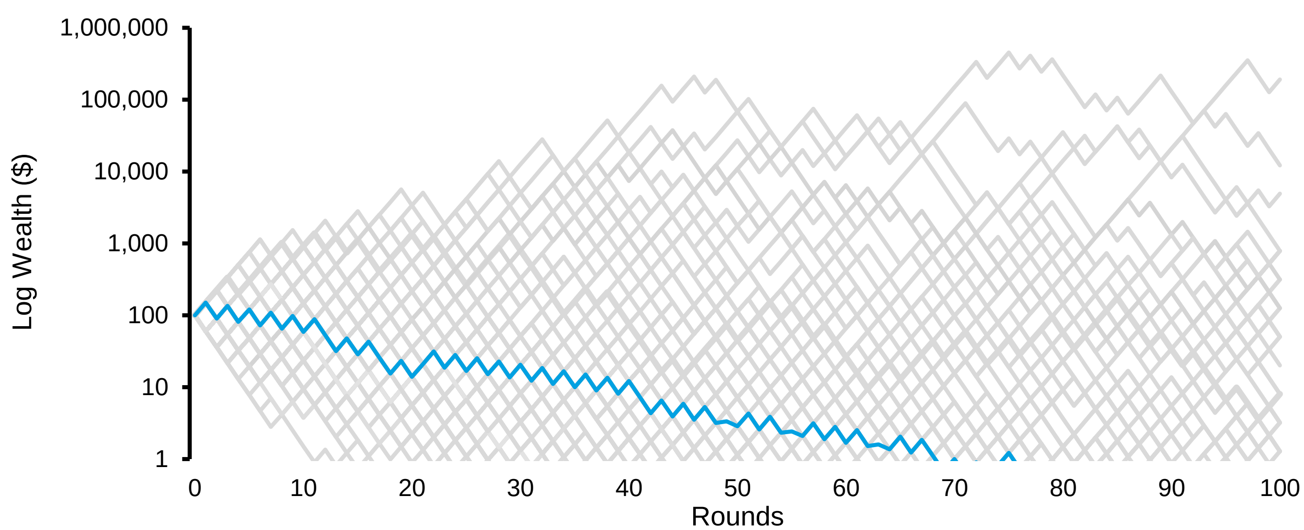
Source: Counterpoint Global.

Let us now consider a process that Ole Peters uses to illustrate a non-ergodic process. You flip a fair coin that increases wealth 50 percent when it comes up heads and decreases it 40 percent when it comes up tails. If you play with $1, the game also has an expected value of $0.05 ([0.50 × $0.50] + [0.50 × -$0.40] = $0.05) and expected wealth of $1.05 ([0.50 × $1.50] + [0.50 × $0.60] = $1.05).

Again we assume that 100 people, each starting with $1, play the game at the same time. About one-half of the ensemble will land on heads and end up with $75 [$50 + (50 × $0.50) = $75], and the other half on tails and end up with $30 [$50 + (50 × -$0.40) = $30]. The expected wealth of the ensemble is $105 [$75 + $30 = $105].

But the experience of one person playing 100 rounds is very different because the geometric mean multiplier is less than 1 ($\sqrt{1.5 \text{ x } 0.6} \approx 0.95$). Exhibit 11 shows that the median wealth goes down as this game is played over time. The average wealth also declines in the long run. The process is non-ergodic because the payoffs are multiplicative. The ensemble and time averages are totally different.

Exhibit 11: Median Wealth Change in a Non-Ergodic Game



Source: Counterpoint Global based on Ole Peters.

Experience tells us that the investment returns for the stock market and portfolios of stocks are non-ergodic. Capital accumulation is a multiplicative process, which means that understanding geometric averages, risk management, and portfolio construction are all essential for compounding wealth.

That life outcomes are non-ergodic also helps explain the value of buying insurance.[71] A personal setback such as losing a home to a fire or a costly medical treatment can substantially damage an individual's wealth and wealth trajectory. Insurance improves the time average growth rate for the insured because the reduction in wealth from paying premiums is more than offset by the prevention of financial disaster. Insurance is attractive from the insurer's point of view because spreading risk among a population makes ensemble averages relevant.

The Kelly criterion makes clear the importance of thinking about geometric means when investing. But it provides two additional lessons for investors, even for those who do not apply the criterion formally.

Pretend that you can participate in a game with a biased coin where heads show up 60 percent of the time. The payoffs are even money, which means if you bet $1 and win you get another $1, and if you are wrong you lose your $1. You start with a $25 bankroll and can wager any amount of your available bankroll for each round. What betting strategy will allow you to achieve the highest probability of the most money after 100 rounds?

Victor Haghani and Richard Dewey, professional investors, presented this game to 61 participants, including college students studying finance and young professionals at financial firms. Haghani and Dewey promised to pay them their final balance in cash (capped at $250).[72] The participants played 119 rounds on average, and heads turned up 59.6 percent of the time.

Their exercise showed that this group did not know how to approach the problem even though the probabilities and payoffs were set. About one-third of the participants lost money and an astounding 28 percent went bust. Twenty one percent earned the maximum amount, which means about half of the players earned an amount below the maximum but above zero. The average ending bankroll was $75.

The Kelly criterion provides an optimal way to engage in this game. Betting nothing makes no sense because the proposition has a positive expected value of $0.20 for every dollar bet on heads ([0.60 × $1] + [0.40 × -$1] = $0.20). But betting it all is also foolish because you lose the entirety of your money if tails appears.

We can present the Kelly criterion multiple ways, but a common approach to calculate the fraction of the bankroll to bet, *f*, is as follows:

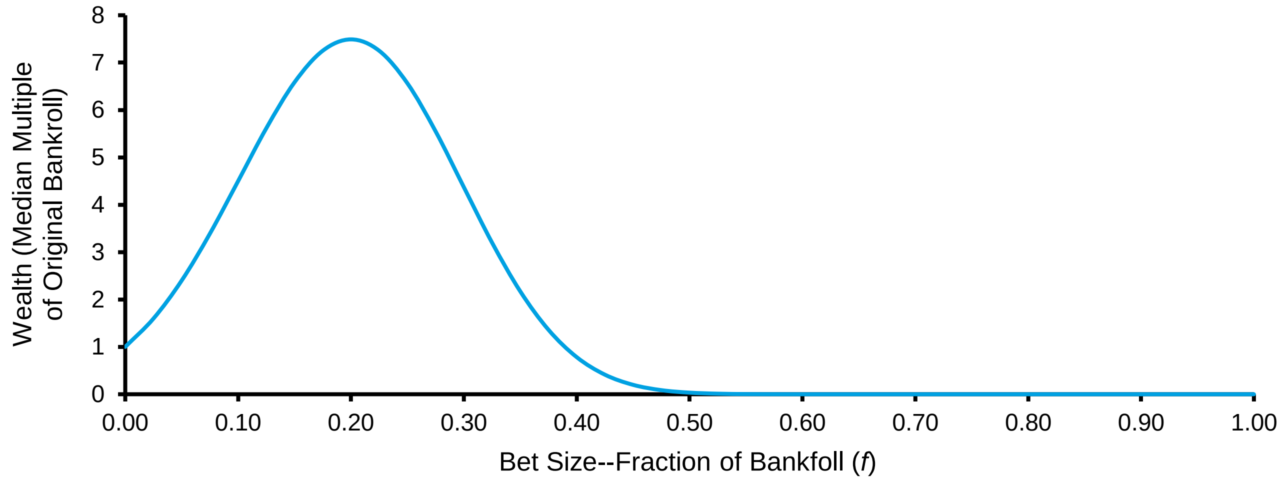
$$f = \frac{\text{Edge}}{\text{Odds}}$$

Edge is the expected value of the proposition, or $0.20 in this case. Odds is how much you win if you win, which is $1.00. So Kelly says the optimal bet size is 20 percent of your bankroll.[73]

The equation highlights the first lesson, which is never bet when you do not have an edge. If edge is zero, *f* is zero. In other words, generating excess returns requires having a well-grounded view that is different than what the market has priced into an asset. This is consistent with seeking opportunities where the expected value is different than the price, as well as ensuring a margin of safety. A corollary is that more attractive investment opportunities should be larger positions in a portfolio than less attractive ones.

What happens if you select a strategy other than what Kelly prescribes? Exhibit 12 shows the results of 1,000 simulations of various proportional betting strategies over 100 rounds. The x-axis is the proportion of the bankroll bet in each round and the y-axis is the median multiple of the initial bankroll after all of the rounds.

Exhibit 12: The Kelly Criterion Reveals the Optimal Betting Strategy



Source: Counterpoint Global.

The chart shows that 20 percent is the proportional bet size that leads to the greatest wealth and that betting too little fails to take advantage of the available edge.

But the other crucial lesson from the Kelly criterion is that it is possible to wager too much. Beyond a certain point bigger bets lead to less, not more, return. Several studies suggest that some of the largest failures of hedge funds in history were the consequence of overbetting.[74]

A handful of academics have claimed that some of the best investors of all time, including John Maynard Keynes, Warren Buffett, George Soros, and Edward Thorp, employed a version of the Kelly criterion.[75] Thorp, trained as a mathematician, is among the most articulate and successful users of the Kelly criterion. He developed a system of card counting in blackjack, a casino game, that allowed players to know when the odds were in their favor. He then paired the edge gained from his approach with the Kelly criterion to optimize potential winnings.[76]

Thorp also successfully applied the Kelly criterion to the stock market. In November 1969, Thorp co-founded a hedge fund that was eventually called Princeton Newport Partners. From the time of its founding through May 1998, the fund produced a compound annual return of approximately 20 percent, a standard deviation of just 6 percent, and a correlation with the stock market of near zero.[77]

The Kelly criterion undoubtedly offers valuable lessons that some investors have used successfully. But the approach has had plenty of vocal critics as well.[78] Some of the concerns include the following:

- **Large estimation risk.** Our simple examples assume that we know the probabilities and payoffs and can confidently calculate arithmetic and geometric means. The ability to forecast distributions in the real world falls on a continuum. A Kelly strategy is difficult to use without reliable inputs.
- **High volatility.** A full allocation prescribed by Kelly, even in simple games such as the biased coin, can result in a wild ride of volatility to the point of terminal wealth. That volatility deters investors, and those who rely on those investors as agents, from staying the course. Practitioners commonly use fractional Kelly allocations to dampen that volatility.
- **The long run.** A Kelly system offers the highest probability of the most wealth in the long run. But some individuals may need access to funds in the near term and therefore are unable to invest for the long haul. As a result, fluctuations in the short term can blunt the benefits of compounding in the long term.

- **Dynamic opportunity set.** The system assumes the investment probabilities and payoffs are relatively constant and that the opportunity set is sufficient to support a growing asset base. It is harder to apply the system when investment payoffs and opportunities are in flux.
- **Portfolio construction.** The Kelly criterion is effective for a single, repeated opportunity. But the application is vastly more complex for portfolios with a mix of opportunities. Optimizing for Kelly while constructing a portfolio with multiple assets and varying degrees of correlation is a challenge.
- **Some chance of disappointment.** Maximizing the geometric mean increases the chance of ending up with more wealth than other strategies but by no means guarantees it. There is always a small chance an investor will do poorly and violate his or her utility function.

The main takeaway from this discussion is that there is a crucial difference between selecting the opportunity with the highest expected value for one period and reinvesting returns in opportunities over multiple periods. Mean/variance optimization is built for the former and geometric mean maximization works for the latter.

The lesson from ergodicity economics is that the experience of many (ensemble) is often irrelevant for the experience of one (time average). You lead just one life and many of your results are path dependent, where past outcomes influence future outcomes. Buying insurance makes sense because avoiding a disastrous result is vital in a multiplicative process.

The Kelly criterion is a formal way to select investments and to size them appropriately. We observe that few members of the fundamental equity investment community use Kelly. But its lessons are relevant, including always seek edge, make your best investments your biggest positions, and never bet too much.

The Implications of Volatility Drag

We noted that the difference between an arithmetic and geometric average is the volatility of returns. This is called volatility drag and can create a large gap between the two measures of average.

There is no volatility drag when there is no volatility. For instance, zero coupon bonds, which do not pay interest but start at a deep discount and accrete value at a steady rate until maturity, are an example of an asset where the arithmetic and geometric average returns are the same.

Volatility drag arises because of the compounding effect of gains and losses. Start with $100 and assume you are up 100 percent in year one (to $200) and down 50% in year 2 (to $100). The arithmetic average is 25 percent ([1.00 + -0.50] ÷ 2 = 0.25) and the geometric average is zero ($[\sqrt{2 \times 0.50}] - 1 = 0$). Geometric returns are key over time since it is capital accumulation that builds wealth.

Here is a common rule of thumb:

$$\text{Arithmetic mean} - \frac{\text{Variance}}{2} \cong \text{Geometric mean}$$

Variance is standard deviation squared. Standard deviation is a measure of dispersion around the average. We go back to the example of S&P 500 returns for the 20 years through 2024 to illustrate the calculation. The index's arithmetic return was 11.9 percent and the standard deviation was 17.3 percent. The variance was therefore 3 percent (0.173^2), and half of the variance is 1.5 percentage points. These figures suggest a geometric mean of 10.4 percent (0.119 – 0.015 = 0.104), which happens to be identical to the realized value.

Exhibit 13 shows the stocks in the S&P 500 with the highest and lowest geometric returns from 2005 to 2024. Our sample includes only those stocks that traded the whole time. Take note of the difference between the arithmetic and geometric average annual returns. Lennox International, a provider of climate control solutions, and Alphabet, a technology company that owns Google, had identical geometric returns (20.2 percent per year) but Lennox's average annual arithmetic return, 23.5 percent, was quite a bit lower than Alphabet's 27.5 percent.

The other feature of this exhibit worth highlighting is the maximum drawdown, the largest decline from peak to trough based on intraday prices, that each stock had over the 20 years. The average drawdown for the best performing stocks was 69 percent, and a handful experienced drawdowns of more than 75 percent. Reaching the peak of total shareholder returns almost always requires going through a valley.

Exhibit 13: Volatility Drag of Stocks with Highest and Lowest Returns in the S&P 500, 2005-24

		TSR, Annual Average		Standard	Volatility	Max
Name		Arithmetic	Geometric	Deviation	Drag	Drawdown
S&P 500		**11.9%**	**10.4%**	**17.3%**	**1.5%**	**-55.3%**
Top 20						
1	NVIDIA	65.2%	39.2%	87.3%	26.0%	-85.5%
2	Netflix	56.5%	36.5%	86.0%	20.0%	-82.7%
3	Apple	41.8%	32.0%	52.3%	9.8%	-61.5%
4	Booking Holdings	40.7%	30.7%	59.6%	9.9%	-68.7%
5	Texas Pacific Land Corporation	36.0%	28.4%	45.3%	7.6%	-74.3%
6	Monster Beverage	38.9%	28.0%	74.8%	10.9%	-70.0%
7	Intuitive Surgical	42.4%	26.9%	73.7%	15.5%	-76.4%
8	Amazon.com	37.2%	25.8%	56.9%	11.4%	-65.7%
9	Salesforce	33.4%	24.4%	47.0%	8.9%	-72.3%
10	Deckers Outdoor	38.8%	24.3%	61.7%	14.5%	-77.6%
11	Regeneron Pharmaceuticals	31.4%	24.3%	50.6%	7.2%	-58.9%
12	Monolithic Power Systems	32.0%	24.0%	44.6%	8.0%	-76.1%
13	Tyler Technologies	28.6%	23.6%	36.0%	5.0%	-49.6%
14	Fair Isaac Corporation	27.5%	22.3%	36.1%	5.3%	-79.9%
15	Old Dominion Freight Line	25.8%	22.0%	31.2%	3.9%	-53.9%
16	O'Reilly Automotive	23.6%	21.9%	21.0%	1.7%	-48.5%
17	Domino's Pizza	29.6%	21.7%	41.0%	7.9%	-92.7%
18	Quanta Services	24.6%	20.3%	32.7%	4.4%	-70.2%
19	Lennox International	23.5%	20.2%	28.9%	3.4%	-55.0%
20	Alphabet	27.5%	20.2%	41.9%	7.3%	-66.9%
	Top 20 Average	**35.3%**	**25.8%**	**50.4%**	**9.4%**	**-69.3%**
Bottom 20						
1	American International Group	5.9%	-11.4%	40.7%	17.3%	-99.6%
2	Citigroup	1.3%	-7.1%	37.2%	8.4%	-98.3%
3	Walgreens Boots Alliance	0.4%	-4.3%	30.0%	4.6%	-91.7%
4	Paramount Global Class B	4.4%	-2.5%	37.4%	6.9%	-96.1%
5	APA Corporation	4.5%	-2.5%	41.0%	6.9%	-97.5%
6	Carnival Corporation	5.6%	-2.1%	41.6%	7.7%	-91.6%
7	Viatris, Inc.	3.3%	-0.8%	31.4%	4.1%	-89.0%
8	KeyCorp	5.6%	-0.1%	33.9%	5.7%	-89.0%
9	PG&E Corporation	3.2%	-0.1%	23.9%	3.3%	-95.0%
10	MGM Resorts International	10.6%	0.0%	42.4%	10.5%	-98.2%
11	Newmont Corporation	4.6%	1.0%	29.4%	3.6%	-78.7%
12	Regions Financial Corporation	7.0%	1.1%	33.6%	6.0%	-94.0%
13	Mohawk Industries	6.2%	1.3%	30.0%	4.8%	-84.3%
14	Ford Motor Company	19.1%	1.4%	86.7%	17.7%	-93.2%
15	Devon Energy Corporation	10.5%	1.4%	53.7%	9.1%	-96.3%
16	Huntington Bancshares	6.5%	1.5%	33.1%	5.0%	-96.1%
17	AES Corporation	6.5%	1.6%	30.6%	4.9%	-80.2%
18	Bank of America Corp	9.4%	1.9%	39.2%	7.5%	-95.4%
19	Intel Corporation	8.3%	2.0%	35.8%	6.3%	-73.3%
20	Zimmer Biomet Holdings	4.5%	2.1%	23.1%	2.4%	-67.5%
	Bottom 20 Average	**6.4%**	**-0.8%**	**37.7%**	**7.1%**	**-90.2%**

Source: FactSet and Counterpoint Global.
Note: Based on companies in the S&P 500 as of 12/31/2024 that traded for the entire period; TSR=total shareholder return.

One extreme example of volatility drag is the GraniteShares 3x Long MicroStrategy Daily ETP (exchange-traded product). This is a security that seeks to provide total return exposure equal to three times the daily performance of Strategy (formerly called MicroStrategy Inc.), a software company that is a large holder of the cryptocurrency Bitcoin.[79] For example, if Strategy's stock goes up 5 percent in a day, the security is designed to rise 15 percent (excluding slippage from tracking error, fees, and "eventual market disruption events").

In 2024, Strategy shares were up 358.5 percent and the GraniteShares 3x ETP, which trades on the London Stock Exchange, were *down* 47.6 percent. At a high level, the reason is that the leverage factor is reset every day. After days when the stock has gone up, the fund increases its exposure to maintain the three times leverage ratio. And when the stock goes down it reduces its exposure. This "buy-high" and "sell-low" feature creates the huge gap between the underlying asset and the fund.[80]

GraniteShares is clear about these risks in its product material, including the point that holding for longer than one day will create a return gap between the fund and Strategy's stock. But it does not seem natural that a fund aiming to offer returns three times those of the underlying stock can go down a lot over a period when the stock goes up a lot.

Volatility drag and drawdowns are reasons it is psychologically difficult to deal with probabilistic systems. There are various challenges, including failing to accurately assess probabilities and payoffs, streaks of losses despite making positive expected value investments, and the practical and mental challenge of drawdowns.

The Psychology of Dealing with Probabilities and Payoffs

The psychology of surprise is the study of how we react when outcomes differ meaningfully from expectations. You can assume that investors who lament that an adverse outcome was a "20-sigma event" or a "perfect storm" misunderstand the underlying probabilities and payoffs.

One vivid example is the stock market crash in 1987. Roger Lowenstein, a journalist, summarized some academic research on the crash: "Economists later figured that, on the basis of the market's historical volatility, had the market been open every day since the creation of the Universe, the odds would still have been against its falling that much in a single day. In fact, had the life of the Universe been repeated *one billion times*, such a crash would still have been theoretically 'unlikely'."[81] The goal is to have sufficient humility when dealing with unknown unknowns and to act accordingly.

Loss aversion is the idea that we suffer losses more than we enjoy gains of comparable size. Exhibit 3 shows it visually. Daniel Kahneman, who won the Nobel Prize in Economics despite being a psychologist, suggested that "loss aversion is certainly the most significant contribution of psychology to behavioral economics."[82]

Academic research has found that the average loss aversion coefficient is about 2.0 and the median is 1.7.[83] That means the negative utility of losing $1 is twice the positive utility of gaining $1. But it is important to acknowledge that the loss aversion coefficient for a population is distributed rather than uniform.

Loss aversion coefficients also vary by age and gender. Loss aversion tends to follow the shape of a "U," high for young people aged 18-24, troughing in the age range of 35-44, and again rising for adults over 55 years old. Women also have a loss aversion coefficient that is consistently higher, although only modestly so, than that of men.[84]

The practical consequence is that two people facing the same economic proposition may consider it differently, and two people living through the same series of outcomes may react differently. The heterogeneity of loss aversion coefficients is consistent with varying utility functions.

Another important consideration is that our individual loss aversion coefficient, no matter what it is at baseline, can change based on our recent financial experience. Specifically, we tend to suffer losses more after having realized losses.[85] That shift in aversion can alter decision-making.

To illustrate the point, scientists created an investment game that compared the results of normal participants, recruited from the local community, with participants who had brain damage.[86] Importantly, those with brain damage had normal intelligence and the parts of their brains that dealt with logic and reasoning were intact. The damage made it so these participants did not have normal feelings of fear or anxiety.

Each person was endowed with $20 at the start of the game. In each round the players had to decide whether or not to invest one dollar, and the game would last 20 rounds. If the player did not play, they would keep their dollar and move on to the next round. If they played, the experimenter flipped a fair coin and paid $2.50 for tails and nothing for heads. The scientists created an incentive to end up with as much money as possible by promising a gift certificate in the amount that the participant won.

The game is analytically straightforward, with a certain value of $1 to not play and an expected value of $1.25 to play (0.50 × $2.50 = $1.25). The ideal strategy is to play every round.

The scientists tallied the results and found that the participants with brain damage ended up with 13 percent more money, on average, than those with normal brains ($25.70 versus $22.80).

Overall, the patients with brain damage played in 45 percent more rounds than the normal players did, and they invested in rounds following a loss at a rate double that of the normal players.

The pattern of play is telling. All of the participants played at a high rate in the first five rounds. This shows that everyone understood that the expected value was positive for each round. But as the game went on, normal people chose to play fewer rounds after having suffered losses. Loss aversion kicked in.

The patients with brain damage, immune from fear and anxiety, played at a high rate throughout the experiment. Brain damage, while debilitating in day-to-day life, spared them the sense of loss aversion and allowed them to focus on expected value.

Baba Shiv, a professor of marketing and one of the scientists running the study, observed that the normal participants "know the right thing to do is invest in every single round, but when they actually get into the game, they just start reacting to the outcomes of previous rounds."[87]

Stop a moment to consider the implication. Individuals are willing to pass over positive expected value propositions after having suffered losses. In periods following large losses in the stock market, such as March 2009, the difficulty is not finding investment opportunities with attractive expected values but rather overcoming the aversion to losing more money.

The certainty and magnitude of payoffs can vary for investment opportunities. How investment alternatives are presented can alter how people choose between them. Importantly, individuals often show preferences that disagree with expected utility theory.

For example, take a look at the two opportunities in exhibit 14 and select the choice from each that you prefer:

Exhibit 14: The Allais Paradox

Opportunity 1			Opportunity 2		
Choice	Probability	Payoff	Choice	Probability	Payoff
A	100%	$1,000,000	**C**	89%	$0
				11%	$1,000,000
or			or		
B	89%	$1,000,000	**D**	90%	$0
	1%	$0		10%	$5,000,000
	10%	$5,000,000			

Source: Based on Maurice Allais and Ole Hagen, eds., Expected Utility Hypotheses and the Allais Paradox *(Dordrecht, Holland: Springer Science + Business, 1979), 25-145.*

Maurice Allais, a physicist and economist who won the Nobel Prize in Economics, showed these opportunities to participants and found that they generally selected choice A from the first opportunity and D from the second one. Selecting A and C, or B and D, is consistent with theory. But picking A and D, certainty in the first case and higher expected value in the second, demonstrates inconsistent preferences and violates the axiom of independence.[88]

Time also plays a significant role in the psychology of investing. Richard Thaler, a winner of the Nobel Prize in Economics, and Shlomo Benartzi are behavioral economists who introduced the concept of "myopic loss aversion."[89] They combine loss aversion with myopia, or nearsightedness, to explain why some short-term oriented investors may suffer more from loss aversion than long-term investors do.[90]

Here is how it works. The stock market tends to go up over time because investors expect a positive return to compensate them for deferring consumption. But returns in the short term are negative some percentage of the time. For example, using past results as a guide, we estimate the probability of a positive gain to be about 55 percent for 1 day, 59 percent for 1 week, 63 percent for 1 month, and 73 percent for 1 year. An investor who looks at her portfolio frequently is more likely to see losses, and suffer from loss aversion, than the investor who looks at her portfolio infrequently.

The implication is that valuation depends in part on time horizon, as short-term investors will demand a higher risk premium to overcome their loss aversion than will long-term investors. A slew of follow-up research suggests that myopic loss aversion is relevant for individual and institutional investors.[91]

Psychology also enters into an assessment of investment process. Results in the investment industry, similar to any field with payoffs and probabilities, has a large dose of luck in the short term. That means that someone can make good decisions and have bad outcomes.

Difficult periods of returns, which are inevitable as part of the process of building a successful long-term investment record, cast doubt on the ability of an investment process to identify opportunities with attractive expected value. This creates a psychological challenge of determining whether disappointing results are the consequence of a good process with normal variance, which is acceptable, or a bad process, which is not.

Finally, we return to the challenging effect of large drawdowns. Hendrik Bessembinder, a professor of finance, identified the stocks of companies that created the most wealth in the last century, including Apple, Microsoft, and Amazon. Bessembinder examined the characteristics of the greatest wealth creators and noted that all of them suffered from large drawdowns along the path to success.[92]

For example, Amazon's stock had a compound annual growth rate of 33.5 percent from the price of its initial public offering to the end of 2024 and created more than $2 trillion in wealth net of Treasury bill returns. But from December 9, 1999 to October 1, 2001, the stock suffered a drawdown of 95 percent based on intraday prices.

Large drawdowns create three challenges. The first is that it is hard for an institutional investor to hold a stock through a large drawdown due to concerns of being wrong and skeptical queries from clients. Second, as exhibit 13 reveals, some stocks have large drawdowns and do not recover. Finally, drawdowns in institutionally-managed portfolios commonly lead to investor outflows, forcing a portfolio manager to sell positions when they are down. This is important because academic research shows that institutional investors select stocks effectively when based on valuation but that decisions induced by investor flows tend to be deleterious to fund results.[93]

Psychology is important because it helps explain what we expect to see, how we react to losses, and how our preferences can change based on our recent experience. We now turn to the probability and payoff characteristics of various asset classes.

Investing in Various Asset Classes

There are multiple asset classes within equities, including public equities, buyouts, and venture capital. Each has its own profile of probabilities and payoffs, which you can think of as the raw material for constructing a fund. The differences are relevant for the portfolio managers and investors in each asset class.

Richard Grinold, a former Global Director of Research at Barclays Global Investors, developed what he called the "fundamental law of active management:"[94]

$$\text{Information ratio} = \text{Information coefficient} \times \sqrt{\text{Breadth}}$$

The equation says that excess return (information ratio) equals skill (information coefficient) times the opportunity set (square root of breadth). More formally, the information coefficient is the correlation between forecasts and outcomes, and breadth is the number of independent opportunities for excess returns in a specified period.

Ronald van Loon, a portfolio manager at BlackRock, further breaks down information coefficient into batting average, which is "the number of winning decisions as a proportion of total decisions," and slugging ratio, "the average return of the wins over the negative of the average return of the losses." He also worked out a way to deal with distributions of asset returns with fat tails.[95]

Skill shows up in the batting average and slugging ratio. Portfolio construction, how investments are weighted in the portfolio, is also relevant. The pattern of returns for the underlying assets plays a substantial role in how a manager reveals his or her skill.

The best way to think about opportunity set, or breadth, is dispersion.[96] The intuition is straightforward. We can use stocks as an example. If the expected returns for all relevant stocks are very similar it is hard for an investment manager to distinguish him or herself. If the expected returns are highly dispersed, a skillful manager produces excess returns by selecting the ones that go up and avoiding the ones that go down.

Breadth can also incorporate the concept of access. This is especially true in private markets. For example, the entrepreneur behind a promising startup may seek funding from venture capital but might consider only a small amount of capital from a few investors. Venture capitalists who have access to the best deals have a large advantage relative to those who do not.

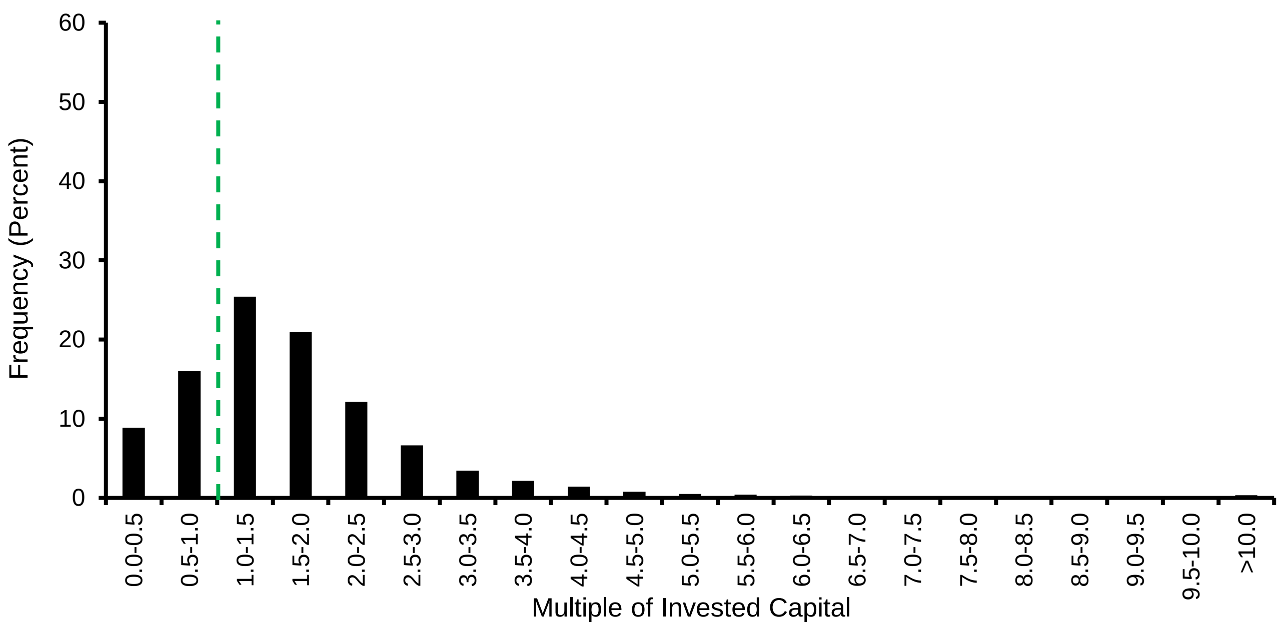
Understanding the dispersion of the returns of the available investment opportunities is crucial. Think of it this way: the dispersion of the fund returns in each asset class will mirror the dispersion of the investment returns. Examining how the pool of potential investments performs gives a sense of the characteristics of the asset class.

We now look at the actual distribution of returns that comprise the opportunity set for public equities, buyouts, and venture capital. To provide some grounding, we estimate that the assets under management (AUM) for active and index funds that manage U.S. public equities are at least $40 trillion at the end of 2024. The AUM was $2.7 trillion for the buyout industry and $1.3 trillion for the venture capital industry in the U.S. in mid-2024, according to PitchBook, a financial data company that tracks private markets.[97]

Mutual funds generally own 50-100 stocks and there are about 4,000 public companies in the U.S. Buyout funds normally hold between 10-20 companies and control around 12,000 companies in total. Venture capital funds commonly make 10-50 investments per fund, with funds that invest in later stages on the low side and those that invest in earlier stages on the high side, and own more than 58,000 companies in aggregate.[98] There are roughly 5.5 million companies in the U.S.[99]

Exhibit 15 shows about 34,000 observations of 5-year returns, measured as multiples of invested capital at the beginning of the period, for stocks in the Russell 1000. We collected 35 increments of 5-year returns from year-end 1985 through year-end 2024. We selected five years because that is similar to the historical average holding period for a company in a private equity portfolio, with VC slightly longer and buyouts slightly shorter than 5 years on average. Note that 25 percent lose money, the modal outcome is a gain of between 1 and 1.5 times, and there are few extreme values.

Exhibit 15: Distribution of Returns for Public Equities



Source: FactSet and Counterpoint Global.
Note: Based on companies in the Russell 1000.

If you extend the time horizon, the cumulative impact of compounding leads to even more skewed results. For example, Bessembinder shows that about 60 percent of the stocks around the world have earned returns below those of Treasury bills and roughly 2 percent of stocks have created about 90 percent of the aggregate wealth.[100]

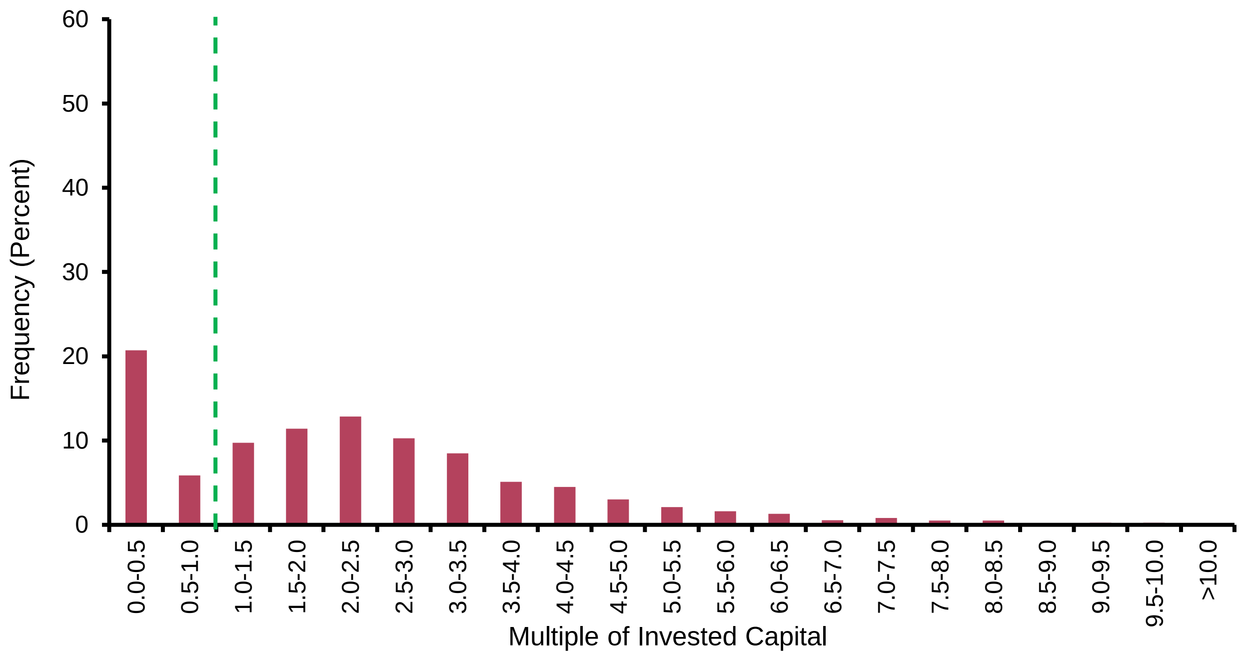
Bessembinder collaborates with other researchers to show that long-term returns for mutual funds follow a similar pattern.[101] Both findings are consistent with the idea that the returns for stocks are non-ergodic.

Exhibit 16 shows more than 15,000 observations of returns, measured as multiples of initial invested capital, for global buyout deals. Most of the returns are from transactions done from the mid-1990s to 2018. Twenty-seven percent lose money, the modal outcome is a loss of 50 to 100 percent of invested capital, and the tails are fatter than those for public equities.

Academics developed a measure called "public market equivalent" (PME) to make a direct comparison between returns in private versus public markets. PME is generally reflected as a ratio between private equity and public market returns, with a ratio above 1.0 suggesting relative outperformance.[102]

Studies show that buyout funds have generally had PMEs in excess of 1.0, although this finding is not without challenge.[103] This result shows that an asset class with a lower batting average can have a higher return than one with a higher batting average because of the pattern of payoffs. Venture capital is a more extreme example.

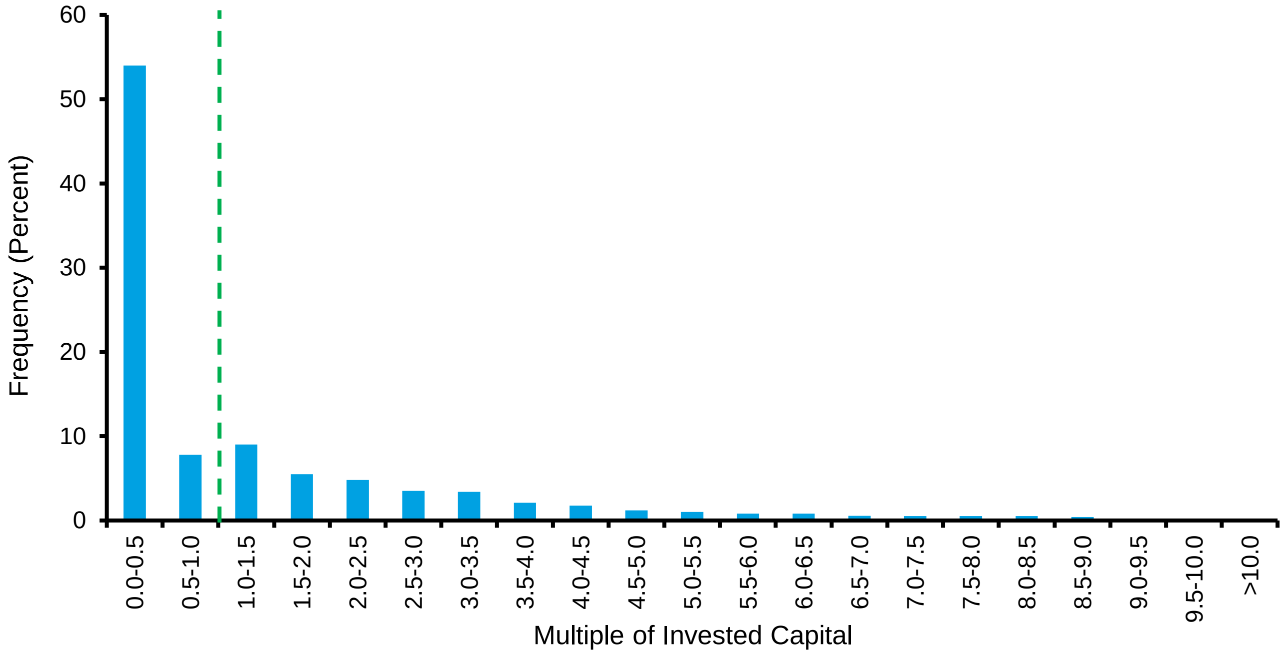
Exhibit 16: Distribution of Returns for Buyout Deals



Source: Based on Gregory Brown, Robert S. Harris, Wendy Hu, Tim Jenkinson, Steven N. Kaplan, and David Robinson, "Private Equity Portfolio Companies: A First Look at Burgiss Holdings Data," SSRN Working Paper, *March 3, 2020.*

Exhibit 17 shows in excess 31,000 observations of returns, measured as multiples of invested capital at the beginning of the period, for global venture capital deals. These results are also from the mid-1990s to 2018. Sixty-two percent lose money and more than one-half of all deals lost 50 to 100 percent of invested capital. The offset is that the tails are much fatter than those for public equities or buyouts.

Exhibit 17: Distribution of Returns for Venture Capital Deals

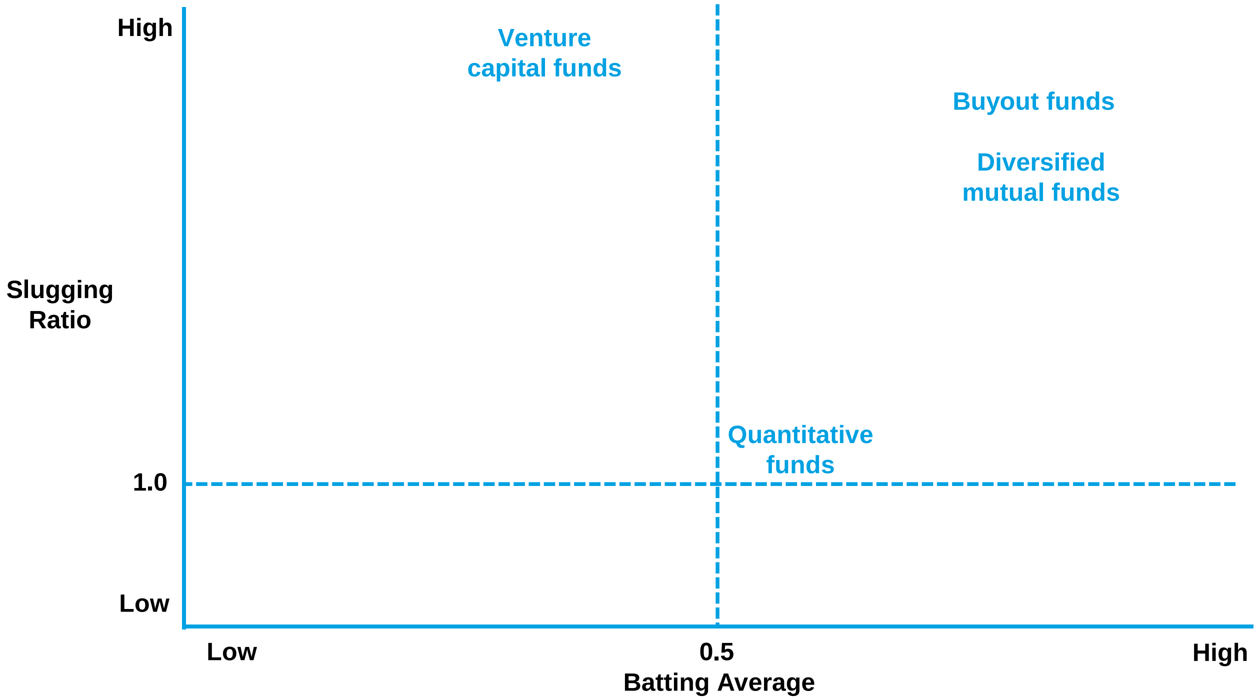


Source: Based on Gregory Brown, Robert S. Harris, Wendy Hu, Tim Jenkinson, Steven N. Kaplan, and David Robinson, "Private Equity Portfolio Companies: A First Look at Burgiss Holdings Data," SSRN Working Paper, *March 3, 2020.*

The PMEs for venture capital have also been above 1.0, and higher than buyouts, over time. But the high PMEs in venture have come in bursts, with long stretches of PMEs close to or below 1.0 interspersed with periods of very high returns. Venture capital fund returns show that it is possible to have a batting average below 50 percent and still have satisfactory returns if the slugging ratio is sufficiently high.

Exhibit 18 shows our estimate of the batting average and slugging ratio of a handful of asset classes within equities. There is naturally a lot of variance for funds in each asset class. The point is that there are meaningfully different paths to seeking excess returns. One study compared the returns of equity funds run systematically (e.g., quantitative funds) to those run with human discretion (e.g., diversified mutual funds) and concluded, "systematic and discretionary funds have historically had similar performance after adjusting for volatility and factor exposures."[104]

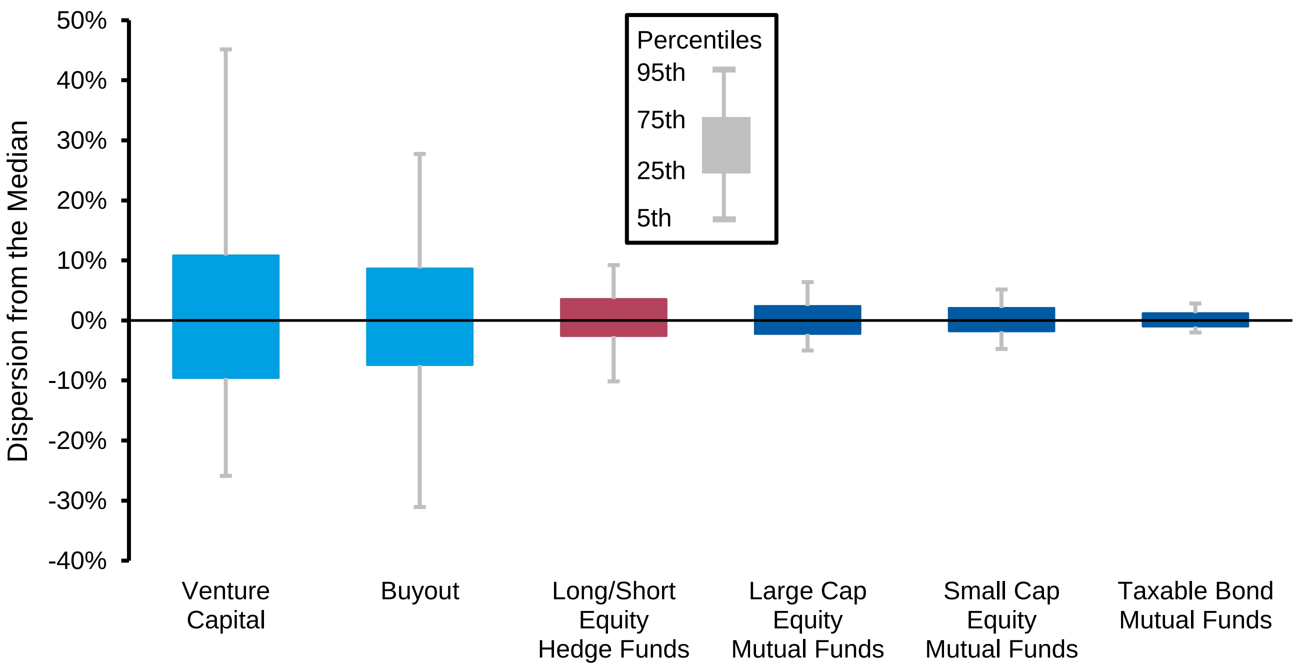
Exhibit 18: Batting Average and Slugging Ratio for Various Equity Asset Classes



Source: FactSet and Counterpoint Global.

Exhibit 19 shows how the dispersion of the opportunity set translates into the dispersion of fund performance by asset class.[105] As the underlying return data would suggest, venture capital funds have the highest dispersion, followed by buyout funds. The dispersion for mutual funds that invest in large capitalization stocks is much lower.

Exhibit 19: Dispersion of Returns for Active Managers in Various Asset Classes



Source: Morningstar Direct, PitchBook, and Counterpoint Global.
Note: Venture capital and buyout: net internal rates of return since inception for vintage years 1980-2018; hedge funds and mutual funds: trailing 5-year annualized returns net of expenses with income reinvested through 12/31/2019.

Here again access is relevant. The returns for the top venture and buyout funds have been very attractive, whereas the performance of the bottom funds has been much worse than that of public equities. Investors who were able to gain access to the funds in the top quintile had a markedly different experience than those exposed to the bottom quintile.

Benchmarks are also important. Public equity investors can invest in an index fund at a low cost whereas comparable benchmarks are not readily available in private markets (hence the development of PME as a measure of comparison). Investors in bonds, an asset class with less volatility than that of equities historically, also run into a benchmark problem.[106]

An appreciation of the probabilities and payoffs, as well as the opportunity set, across various asset classes can be useful to an investor seeking to generate excess returns. The nature of the probabilities and payoffs for the underlying investments also means that what constitutes skill, in terms of batting average and slugging ratio, differs by asset class. Again, it is not how often you are right that matters, it is how much you make when you are right versus how much you lose when you are wrong.

Conclusion

The prime task of an investor seeking to generate excess returns is to find opportunities where there are gaps between price and value. This is commonly called variant perception, or edge. Price is relatively straightforward but assessing value can be a challenge. The common approach is to consider expected value, which is the sum of the products of various payoffs and their associated probabilities. The task then becomes coming up with payoffs and probabilities in a thoughtful manner.

The Babe Ruth effect highlights that it is not only how often you are right that matters (probability) but how much you make when you are right versus how much you lose when you are wrong (payoffs). Venture capital, as an asset class, loses more frequently than it wins. But the gains are so large they offset the losses in the aggregate.

Expected values can be characterized in different ways. With risk, no one knows which outcome will occur but all the possible outcomes can be identified in advance. With uncertainty, both the outcomes and the range of possible outcomes are unknown. And then there is the domain of "unknown, unknowns," where ignorance prevents an assessment of what might happen.

Determining payoffs comes with a number of challenges, including identifying the shape of the distribution, reflecting potential non-linearities, recognizing the relationship between control and reversibility, assessing asymmetries, and acknowledging that exogenous and endogenous risks affect payoffs.

There are recognized approaches to setting probabilities, including frequentist, propensity, and subjective belief. Most forecasts in investing are based on subjective beliefs, which follow the laws of probability but require updating with new information. Probabilities also come with varying degrees of confidence—sometimes a belief can be held but with low confidence. Using probabilities instead of words is essential for clarity of communication and as a basis for feedback and learning.

Best practices in setting payoffs and probabilities include using base rates, applying sensitivity analysis and simulation, and, above all, always insisting on a margin of safety. The margin of safety reflects the size of the gap between price and value and allows for incorrect analysis and bad luck.

Assessing expected value through payoffs and probabilities is very useful, and in decisions beyond one period it is optimal to find the highest expected value for an assumed risk appetite. But the situation changes when the process shifts from arithmetic to multiplicative.

In systems that are ergodic, where the ensemble and time averages are the same, it almost always makes the most sense to pursue the approach with the highest arithmetic mean. In systems that are non-ergodic, where the ensemble and time averages are different, the best approach is usually to find the opportunity with the highest geometric mean.

This is important because markets are largely non-ergodic. The experience of the group is not relevant to an individual who goes through life but once. As a consequence, understanding and integrating an appreciation of the geometric mean of an investment opportunity is central to building wealth in the long term.

The Kelly criterion is an investment guideline based on geometric mean maximization. The Kelly criterion offers two useful lessons even for those who do not use the principle in practice. The first is that every investment opportunity should include edge. The second is that it is possible to bet too much. Sometimes increasing the size of an attractive opportunity leads to a lower, not higher, expected return.

Volatility creates the difference between an asset's arithmetic and geometric returns. This is called volatility drag. Many of the best investments over time are volatile and have large drawdowns.

There are psychological challenges in dealing with probabilistic realms. One example is loss aversion, the idea that we suffer roughly twice as much from losses as we enjoy gains of comparable size. While the coefficient of loss aversion is around two on average, there is a great deal of variation by individual. Perhaps more importantly, our loss aversion coefficients tend to go up after we have suffered losses. This means that people may react differently to a financial opportunity based on the circumstances.

Time horizon is also very important. Markets tend to go up in the long term, but losses are common in the short term. Investors who evaluate their portfolios frequently are more likely to see losses and hence suffer from loss aversion. This means that the appetite for risk depends to some degree on the investor's time horizon.

Excess returns are a function of skill and opportunity set. Skill can be assessed through batting average, how often you make money, and slugging ratio, how much you make when you are right versus how much you lose when you are wrong. Dispersion is a useful way to look at investment opportunities.

The opportunity sets of public equities, buyouts, and venture capital vary substantially. For example, based on the figures we used, 25 percent of public equity investments lost money over 5 years compared to 62 percent of venture capital investments. Offsetting that average is the fact that venture had more very high return investments than did public markets.

One consequence of the variation in opportunity sets is the dispersion of returns for managers in each asset class. Dispersion is the highest in venture capital, followed by buyouts, and then public equities. As a result, access is important. Owning venture capital funds in the top quartile of performance has provided handsome excess returns whereas owning those in the bottom quartile has been a challenge.

Investing is an inherently probabilistic endeavor. The ideas surrounding payoffs and probabilities can help the intelligent investor build a portfolio positioned to generate excess returns.

Please see Important Disclosures on pages 47-49

Appendix: Measuring Probabilistic Forecasts with a Brier Score

The Brier score is a common method used to measure the accuracy of probabilistic forecasts. The score was created by Glenn Brier, a meteorologist, in the 1950s.[107] A basic version of the Brier score measures the square of the forecast error. For binary events, the value is 1 if the event occurs and 0 if it does not. A lower score is better.

Brier's original approach had a scale of 0 to 2 (other versions have a scale from 0 to 1). In this basic case the calculation considers the squared forecast error for both the event and the non-event.

Exhibit 20 is an example based on a meteorologist's forecast for rain over four days. Take Day 2 as an illustration of the calculation. Our meteorologist forecasted a 90 percent chance of rain and, by definition, a 10 percent probability it would *not* rain. It did rain, so mark a "1" in the outcome column below "Rain" and a "0" under "No Rain." Our meteorologist's Brier score for that day was 0.02. ($[0.9 – 1]^2 + [0.1 – 0]^2 = 0.01 + 0.01 = 0.02$). An overall Brier score is the average over multiple forecasts. The meteorologist's Brier score over these 4 days is 0.15.

Exhibit 20: Calculation of a Brier Score

	Rain		No Rain		Brier Score	
Day	**Forecast**	**Outcome**	**Forecast**	**Outcome**	**Calculation**	**Result**
1	30%	0	70%	1	$(0.30\text{-}0)^2 + (0.70\text{-}1)^2$	0.18
2	90%	1	10%	0	$(0.90\text{-}1)^2 + (0.10\text{-}0)^2$	0.02
3	45%	0	55%	1	$(0.45\text{-}0)^2 + (0.55\text{-}1)^2$	0.41
4	100%	1	0%	0	$(1.0\text{-}1)^2 + (0.0\text{-}0)^2$	0.00
					Average	**0.15**

Source: Counterpoint Global.

One nice feature of the scale from 0 to 2 is that random guesses have a Brier score of 0.50. Top forecasters of political, economic, and social outcomes have Brier scores of around 0.20-0.25. The key is to communicate using terms that a Brier score can measure. As Phil Tetlock and Dan Gardner write in their book, *Superforecasting*, "Forecast, measure, and revise: it is the surest path to seeing better."[108]

Endnotes

[1] This is not strictly true. Investors can also participate in arbitrage, the simultaneous purchase (at the lower price) and sale (at a higher price) of an identical asset that locks in a profit. In this case, the distinction between price and value is irrelevant and all that matters is that the prices converge. Practically, arbitrage strategies extend beyond buying and selling the same asset, introducing an element of risk. For example, merger arbitrage involves buying or selling the shares of sellers or buyers in a merger or acquisition, and the risk is that the deal does not close or the initial terms change materially. “Arbitrage costs” typically limit the ability to exploit arbitrage opportunities. These costs include those connected to identifying and verifying mispricing, implementing and executing trades, and financing and funding securities. See Charles M.C. Lee and Eric So, “Alphanomics: The Informational Underpinnings of Market Efficiency,” *Foundations and Trends in Accounting*, Vol. 9, No. 2-3, 2014, 175-206.
[2] Michael Steinhardt, a legendary hedge fund manager, wrote, “I defined variant perception as holding a well-founded view that was meaningfully different from the market consensus . . . Understanding market expectation was at least as important as, and often different from, the fundamental knowledge.” Ed Thorp, a mathematician and extremely successful investment manager, said, “There is a market inefficiency if there is a participant who can generate excess risk-adjusted returns that can be logically explained in a way that is difficult to rebut.” See Michael Steinhardt, *No Bull: My Life In and Out of Markets* (New York: John Wiley & Sons, 2001), 129 and Ed Thorp interview in Jack D. Schwager, *Hedge Fund Market Wizards: How Winning Traders Win* (Hoboken, NJ: John Wiley & Sons, 2012), 217.
[3] Ĺuboš Pástor and Robert F. Stambaugh, “Liquidity Risk and Expected Stock Returns,” *Journal of Political Economy*, Vol. 111, No. 3, June 2003, 642-685.
[4] Here is an example of the challenge of understanding the pattern of payoffs. Joe Peta, who was a trader on Wall Street and has done fine work on analytics for the investment industry, wrote a book called *Moneyball for the Money Set: Using Sports Analytics to Predict the Returns of Portfolio Managers with Startling Accuracy*. Joe worked for one of the largest multi-strategy hedge funds and had access to the performance data of the various portfolio managers, which he analyzed in an effort to assess skill. He found that hit rate, or daily percentage of stocks that outperformed an average of relevant stocks in the sector, was one of the most important signals. He argues that a hit rate above 50 percent is a key to success. This is a reasonable measure of skill *for this strategy*. Peta goes on to dedicate a chapter to “debunking the George Soros narrative.” The setup is that the former chief investment officer of Soros Fund Management and current U.S. Secretary of the Treasury, Scott Bessent, is quoted as saying that Soros’s hit rate was less than 50 percent. Peta goes on to suggest that a hit rate of less than 50 percent was “impossible” given Soros’s track record.
What Peta misses is that there are strategies that do succeed with hit rates below 50 percent, including trend followers. (See Michael Covel, “Trend Following Winners Are Not Lucky Monkeys,” *Active Trader Magazine*.) Peta’s analysis appears valid for the investment approach he studies but is not universally applicable.
[5] Michael J. Mauboussin, *More Than You Know: Finding Financial Wisdom in Unconventional Places—Updated and Expanded* (New York: Columbia Business School Publishing, 2008), 24. For a list of great quotations about batting average, see http://mastersinvest.com/battingaverage.
[6] Steven Crist, *Exotic Betting: How to Make the Multihorse, Multirace Bets That Win Racing’s Biggest Payoffs* (New York: DRF Press, 2006).
[7] Katherine Sayre and Isabella Simonetti, “America Has Fallen in Love With Long-Shot Sports Bets,” *Wall Street Journal*, January 25, 2025 and Flutter Investor Day, Management Presentation and Q&A, New York, September 25, 2024.
[8] Gunjan Banerji, “Wall Street’s Hot New Trade Is Fueling Gambling Addictions,” *Wall Street Journal*, December 23, 2024.
[9] Warren E. Buffett, *Berkshire Hathaway Annual Meeting*, 1989.
[10] Daniel Bernoulli, “Exposition of a New Theory on the Measurement of Risk,” *Econometrica*, Vol. 22, No. 1, January 1954, 23-36. To express the point more formally, for someone who is risk averse, as the expected value increases (x-axis), utility increases at a lesser rate (y-axis), and the resulting curve is concave. Simple equations to capture this include $U(x) = x^{0.5}$ or $U(x) = \log(x)$.

[11] Nassim Nicholas Taleb, *The Black Swan: The Impact of the Highly Improbable, Second Edition* (New York: Random House, 2010), 122-131.
[12] Frank H. Knight, *Risk, Uncertainty and Profit* (Boston: Houghton Mifflin Company, 1921), 233. John Maynard Keynes makes similar points in his book, *A Treatise on Probability,* also published in 1921.
[13] Donald H. Rumsfeld (Secretary of Defense), “Department of Defense News Briefing,” February 12, 2002.
[14] Richard Zeckhauser, “Investing in the Unknown and Unknowable,” *Capitalism and Society*, Vol. 1, No. 2, 2006, Article 5.
[15] Don A. Moore, *Perfectly Confident: How to Calibrate Your Decisions Wisely* (New York: Harper Business, 2020), 8.
[16] Benoit B. Mandelbrot, *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk* (New York: Springer, 1997), 117-125 and Benoit B. Mandelbrot and Nassim Nicholas Taleb, “Mild vs. Wild Randomness: Focusing on Those Risks That Matter,” in *The Known, the Unknown, and the Unknowable in Financial Risk Management: Measurement and Theory Advancing Practice*, Francis X. Diebold, Neil A. Doherty, and Richard J. Herring, eds., (Princeton, NJ: Princeton University Press, 2010), 47-58.
[17] A power law is a relationship between two variables where one varies as a constant power of the other. The slope of the line that best fits the data is the exponent, or “power,” that defines the law.
[18] Taleb, *The Black Swan*, xvii-xviii.
[19] Seneca, translated by Robin Campbell, *Letters from a Stoic* (London: Penguin House, 1969), 178.
[20] Philip Ball, *Critical Mass: How One Thing Leads to Another* (New York: Farrar, Straus and Giroux, 2004), 80-97. Ball takes the term “the grand ah-whoom” from Kurt Vonnegut’s book, *Cat’s Cradle*.
[21] James Surowiecki, *The Wisdom of Crowds: Why the Many Are Smarter Than the Few and How Collective Wisdom Shapes Business, Economies, Societies, and Nations* (New York: Doubleday and Company, 2004).
[22] Peter L. Bernstein, “Risk, Time, and Reversibility,” *The Geneva Papers on Risk and Insurance*, Vol. 24, No. 2, April 1999, 131-139.
[23] Benjamin Graham and David L. Dodd, *Security Analysis* (New York: McGraw Hill, 1934), 66.
[24] Daniel Kahneman and Amos Tversky, “Prospect Theory: An Analysis of Decision under Risk,” *Econometrica*, Vol. 47, No. 2, March 1979, 263-292. More accurately, these conclusions derive from “cumulative prospect theory” (see Amos Tversky and Daniel Kahneman, “Advances in Prospect Theory: Cumulative Representation of Uncertainty,” *Journal of Risk and Uncertainty*, Vol. 5, No. 4, October 1992, 297-323.) Barberis and Huang summarize the advance beautifully: “Under cumulative prospect theory, people evaluate risk using a value function that is defined over gains and losses, that is concave over gains and convex over losses, and that is kinked at the origin; and using *transformed* rather than objective probabilities, where the transformed probabilities are obtained from objective probabilities by applying a weighting function. The main effect of the weighting function is to overweight the tails of the distribution it is applied to. The overweighting of tails does not represent a bias in beliefs; it is simply a modeling device that captures the common preference for a lottery-like, or positively skewed, wealth distribution.” See Nicholas Barberis and Ming Huang, “Stocks as Lotteries: The Implications of Probability Weighting for Security Prices,” *American Economic Review,* Vol. 98, No. 5, December 2008, 2066-2100 and Tobias J. Moskowitz and Kaushik Vasudevan, “Betting Without Beta,” *Working Paper*, May 2, 2022.
[25] Kimberly F. Luchtenberg and Michael J. Seiler, “Do Institutional and Individual Investors Differ in Their Preference for Financial Skewness?” *Journal of Behavioral Finance*, Vol. 15, No. 4, 2014, 299-311.
[26] Barberis and Huang, “Stocks as Lotteries” and Suk-Joon Byun, Jihoon Goh, and Da-Hea Kim, “The Role of Psychological Barriers in Lottery-Related Anomalies,” *Journal of Banking & Finance*, Vol. 114, May 2020, 105786.
[27] Antti Ilmanen, “Do Financial Markets Reward Buying or Selling Insurance and Lottery Tickets?” *Financial Analysts Journal*, Vol. 68, No. 5, September/October 2012, 26-36. Perhaps not surprisingly, Nassim Taleb took issue with Ilmanen’s conclusions. See Nassim Nicholas Taleb, “Do Financial Markets Reward Buying or Selling Insurance and Lottery Tickets?: A Comment,” *Financial Analysts Journal*, Vol. 69, No. 2, March/April 2013, 17-19.
[28] Nassim Nicholas Taleb, “Bleed or Blowup? Why Do We Prefer Asymmetric Payoffs?” *Journal of Behavioral Finance*, Vol. 5, No. 1, 2004, 2-7.

[29] Robert J. Shiller, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review*, Vol. 71, No. 3, June 1981, 421-436 and Richard Roll, "R^2," *Journal of Finance*, Vol. 43, No. 3, July 1988, 541-566.
[30] David M. Cutler, James M. Poterba, and Lawrence H. Summers, "What Moves Stock Prices?" *Journal of Portfolio Management*, Vol. 15, No. 3, Spring 1989, 4-12 and Bradford Cornell, "What Moves Stock Prices: Another Look," *Journal of Portfolio Management*, Vol. 39, No. 3, Spring 2013, 32-38.
[31] Cutler, Poterba, and Summers, "What Moves Stock Prices?" 9.
[32] Jon Danielsson and Hyun Son Shin, "Endogenous Risk," in *Modern Risk Management: A History (*London: Risk Books, 2003), 297-313.
[33] David Spiegelhalter, "Does Probability Exist? Probably Not—But It Is Useful to Act As If It Does," *Nature*, Vol. 636, No. 8043, December 19/26, 2024, 560-563.
[34] Gerd Gigerenzer, *Calculated Risks: How to Know When Numbers Deceive You* (New York: Simon & Schuster, 2002), 26-29.
[35] Sharon Bertsch McGrayne, *The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy* (New Haven: Yale University Press, 2011).
[36] Frank P. Ramsey, "Truth and Probability," *The Foundations of Mathematics and other Logical Essays*, Richard B. Braithwaite, ed. (London: Kegan, Paul, Trench, Trubner & Co., 1931), 156-198 and Annie Duke, *Thinking in Bets: Making Smarter Decisions When You Don't Have All the Facts* (New York: Portfolio/Penguin, 2018).
[37] Here's an example of a problem that requires updating prior beliefs based on new evidence. It comes from Daniel Kahneman, *Thinking, Fast and Slow* (New York: Farrar, Straus and Giroux, 2011), 166:

"A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are told that 85 percent of the cabs in the City are Green and 15 percent are Blue. A witness identified the cab as Blue. The courts tested the reliability of the witness under the circumstances that existed on the night of the accident and concluded that the witness correctly identified each of the two colors 80 percent of the time. What is the probability that the cab involved in the accident was Blue rather than Green?"

The most common answer is 80 percent, likely reflecting the accuracy of the eyewitness, but the correct answer is about 41 percent. The tendency is to place too much weight on the account of the witness and not enough weight on the point that a large majority of cabs in the city are Green.

Here is how you get the answer. You need three quantities to solve for the new probability. First is a prior probability. In this case, the prior probability (x) of a Blue cab getting into an accident would be 15 percent (assuming Green and Blue cabs have an equal chance of an accident). Second, is an estimate of the probability as a condition of the hypothesis being true (y). We have a witness who is 80 percent accurate claiming that a Blue cab was in the accident. Finally, is an estimate conditional on the hypothesis being false (z), which is 20 percent (the complement of 80 percent).

Bayes's Theorem tells us the revised probability =

$$\frac{xy}{xy + z(1\text{-}x)} = \frac{.15 \text{ x } .80}{.15 \text{ x } .80 + .2(1\text{-}.15)} = \frac{.12}{.29} = 41.4\%$$

An easier way to think about this is to use natural numbers. Assume there are 1,000 cabs in the city. The eyewitness, examining all of the green ones, would suggest with 20 percent accuracy that one was in the accident (170) and looking at the blue ones would say with 80 percent accuracy that one was involved (120). So the likelihood is 120/(170 + 120), or 41.4 percent. See Sanjit Dhami, *Principles of Behavioral Economics: Microeconomics & Human Behavior* (Cambridge, UK: Cambridge University Press, 2025), 418-419.
[38] Chetan Dave and Katherine W. Wolfe, "On Confirmation Bias and Deviations From Bayesian Updating," *Working Paper*, March 21, 2003. Notwithstanding individual bias, the impact on the market overall may be modest. See Colin F. Camerer, "Do Biases in Probability Judgment Matter in Markets? Experimental Evidence," *American Economic Review*, Vol. 77, No. 5, December 1987, 981-997.
[39] Philip E. Tetlock, *Expert Political Judgment: How Good Is It? How Can We Know?* (Princeton, NJ: Princeton University Press, 2005).

[40] Jeffrey A. Friedman and Richard Zeckhauser, “Analytic Confidence and Political Decision-Making: Theoretical Principles and Experimental Evidence From National Security Professionals,” *Political Psychology*, Vol. 39, No. 5, October 2018, 1069-1087. For example, in January 2025 it was revealed that research by the Central Intelligence Agency (CIA) suggested that the Covid-19 virus was most likely the result of a lab leak, although it had “low confidence” in its finding. See Michael R. Gordon and Dustin Volz, “CIA Now Favors Lab Leak Theory on Origins of Covid-19,” *Wall Street Journal*, January 25, 2025.
[41] John H. Miller and Scott E. Page, *Complex Adaptive Systems: An Introduction to Computational Models of Social Life* (Princeton, NJ: Princeton University Press, 2007).
[42] Andrew Mauboussin and Michael J. Mauboussin, “If You Say Something Is ‘Likely,’ How Likely Do People Think It Is?” *Harvard Business Review Blog*, July 3, 2018.
[43] For more on this topic, see David Spiegelhalter, *The Art of Uncertainty: How to Navigate Chance, Ignorance, Risk and Luck* (Dublin: Penguin Random House, 2024), 31-56.
[44] “Want Better Forecasting Skills? Silence the Noise,” *Knowledge@Wharton*, November 26, 2019.
[45] There is an important distinction between calibration and “resolution.” Calibration measures the alignment between subjective and objective probabilities. Resolution measures the ability to distinguish between high and low probability events. For example, consider daily forecasts of rain in London, England. If you forecasted a 30 percent likelihood every day for a year, you would appear well calibrated (it rains about 30 percent of the days in one year on average). But that does not help for planning picnics. Scoring high on resolution means accurately predicting “no rain” for 70 percent of the days and “rain” for the other 30 percent.
[46] Sarah Lichtenstein and Baruch Fischhoff, “Training for Calibration,” *Organizational Behavior and Human Performance*, Vol. 26, No. 2, October 1980, 149-171 and Philip E. Tetlock and Dan Gardner, *Superforecasting: The Art and Science of Prediction* (New York: Crown Publishers, 2015), 180-182.
[47] Allan H. Murphy and Harald Daan, “Impacts of Feedback and Experience on the Quality of Subjective Probability Forecasts: Comparison of Results from the First and Second Years of the Zierikzee Experiment,” *Monthly Weather Review*, Vol. 112, No. 3, 1984, 413-423.
[48] Mauboussin, *More Than You Know,* 9-14.
[49] Shreenivas Kunte, “The Herding Mentality: Behavioral Finance and Investor Biases,” *CFA Institute Enterprising Investor*, August 6, 2015.
[50] Etienne Theising, Dominik Wied, and Daniel Ziggel, “Reference Class Selection in Similarity-Based Forecasting of Corporate Sales Growth,” *Journal of Forecasting*, Vol. 42, No. 5, August 2023, 1069-1085.
[51] Amos Tversky and Daniel Kahneman, “Evidential Impact of Base Rates,” in Daniel Kahneman, Paul Slovic, and Amos Tversky eds., *Judgment under Uncertainty: Heuristics and Biases* (Cambridge, UK: Cambridge University Press, 1982), 153-160 and Daniel Kahneman and Amos Tversky “On the Psychology of Prediction,” *Psychological Review*, Vol. 80, No. 4, July 1973, 237-251. To amplify on the latter point, when an activity shows high persistence—a strong correlation between sequential outcomes—then more weight is placed on information and individual input. When persistence is low, more weight is assigned to base rates. The degree of persistence is quantifiable for many measures of corporate performance, such as sales growth, operating profit margins, and return on invested capital.
[52] Aswath Damodaran, *Narrative and Numbers: The Value of Stories in Business* (New York: Columbia Business School Publishing, 2017).
[53] Thomas Graeber, Christopher Ross, and Florian Zimmerman, “Stories, Statistics, and Memory,” *Quarterly Journal of Economics*, Vol. 139, No. 4, November 2024, 2181-2225.
[54] Daniel Kahneman and Amos Tversky, “Intuitive Prediction: Biases and Corrective Procedures,” in Daniel Kahneman, Paul Slovic, and Amos Tversky, eds., *Judgment under Uncertainty: Heuristics and Biases* (Cambridge, UK: Cambridge University Press, 1982), 414-421.
[55] Etienne Theising, “Distributional Reference Class Forecasting of Corporate Sales Growth With Multiple Reference Variables,” *ArXiv*, May 6, 2024.
[56] Warren E. Buffett, “Letter to Shareholders,” *Berkshire Hathaway Annual Report*, 2001.
[57] Michael J. Mauboussin and Dan Callahan, “The Impact of Intangibles on Base Rates,” *Consilient Observer: Counterpoint Global Insights*, June 23, 2021.
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[59] David Aboody, Shai Levi, and Dan Weiss, "Operating Leverage and Future Earnings," *Working Paper*, December 7, 2014 and Huong N. Higgins, "Earnings Forecasts of Firms Experiencing Sales Decline: *Why So Inaccurate?*" *Journal of Investing*, Vol. 17, No. 1, Spring 2008, 26-33.
[60] Aswath Damodaran, "DCF Myth 3.2: If You Don't Look, It's Not There!" *Musings on Markets,* May 23, 2016.
[61] Benjamin Graham, *The Intelligent Investor: The Definitive Book on Value Investing, Third Edition, Updated with new commentary by Jason Zweig* (New York: Harper Business, 2024), 505.
[62] These are terms that Graham used.
[63] Harry Markowitz, "Portfolio Selection," *Journal of Finance*, Vol. 7, No. 1, March 1952, 77-91.
[64] J. L. Kelly Jr., "A New Interpretation of Information Rate," *Bell System Technical Journal*, 1956, 917-926.
[65] William Poundstone, *Fortune's Formula: The Untold Story of the Scientific Betting System That Beat the Casinos and Wall Street* (New York: Hill and Wang, 2005), 200.
[66] In Russian roulette, an individual puts a bullet into one of the six chambers of a revolver, spins the cylinder, places the handgun in harm's way, and pulls the trigger. The weapon fires the bullet one-sixth of the time when the loaded chamber is lined up with the barrel.
[67] Nassim Taleb writes, "[U]nlike a well-defined precise game like Russian roulette, where the risks are visible to anyone capable of multiplying and dividing by six, one does not observe the barrel of reality. Very rarely is the generator visible to the naked eye. One is thus capable of unwittingly playing Russian roulette—and calling it by some alternative 'low risk' name. We see the wealth being generated, never the processor, a matter that makes people lose sight of their risks, and never the losers. The game seems terribly easy and we play along blithely." See Nassim Nicholas Taleb, *Fooled By Randomness: The Hidden Role of Chance in the Markets and in Life* (New York, Texere, 2001), 27-29.
[68] Aaron Brown, *Red Blooded Risk: The Secret History of Wall Street* (Hoboken, NJ: John Wiley & Sons, 2012), 73-99.
[69] Harry M. Markowitz, "Investment for the Long Run: New Evidence for an Old Rule," *Journal of Finance*, Vol. 31, No. 5, December 1976, 1273-1286. For an example of work that is similar to that of Kelly, see Henry Allen Latané, "Criteria for Choice Among Risky Ventures," *The Journal of Political Economy*, Vol. 67, No. 2, April 1959, 144-155.
[70] Ole Peters, "The Ergodicity Problem in Economics," *Nature Physics*, Vol. 15, December 2019, 1216-1221.
[71] Ole Peters, "Insurance as an Ergodicity Problem," *Annals of Actuarial Science*, Vol. 17, No. 2, July 2023, 215-218.
[72] Victor Haghani, and Richard Dewey, "Rational Decision Making under Uncertainty: Observed Betting Patterns on a Biased Coin," *Journal of Portfolio Management*, Vol. 43, No. 3, Spring 2017, 2-8 and Victor Haghani and James White, *The Missing Billionaires: A Guide to Better Financial Decisions* (Hoboken, NJ: John Wiley & Sons, 2023), 15-25. You can find a version of this game at https://elmwealth.com/coin-flip/.
[73] Another simple approach to Kelly for even money bets is $f = 2p - 1$. In this case p = 0.60, to $f = 0.20$ ($[2 \times 0.60] - 1 = 0.20$).
[74] Rachel E.S. Ziemba and William T. Ziemba, *Scenarios for Risk Management and Global Investment Strategies* (Chichester, UK: John Wiley & Sons, 2007), 95-122. Leverage commonly plays a role in overbetting. In his paper, "Optimal Leverage from Non-Ergodicity," Ole Peters considers optimal leverage assuming non-ergodicity.
[75] Oliver Gergaud and William T. Ziemba, "Great Investors: Their Methods, Results and Evaluation," in William T. Ziemba, *Great Investment Ideas* (Hackensack, NJ: World Scientific, 2017), 175-212.
[76] Edward O. Thorp, *Beat the Dealer: A Winning Strategy for the Game of Twenty-One* (New York: Blaisdell Publishing Company, 1962).
[77] Edward O. Thorp, "The Kelly Criterion in Blackjack Sports Betting and the Stock Market," in Leonard C. MacLean, Edward O. Thorp, and William T. Ziemba, eds., *The Kelly Capital Growth Investment Criterion: Theory and Practice* (Hackensack, NJ: World Scientific, 2011), 823.
[78] See Paul A. Samuelson, "The 'Fallacy' of Maximizing the Geometric Mean in Long Sequences of Investing or Gambling," *Proceedings of the National Academy of Sciences*, Vol. 68, No. 10, October 1971, 2493-2496; Rubinstein, Mark, "No 'Best' Strategy for Portfolio Insurance," *Letter to the Editor in Financial Analysts Journal*, Vol. 43, No. 6, November/December 1987, 77-80; and Zeckhauser, "Investing in the Unknown and Unknowable."
[79] The reality is a little more complicated. GraniteShares 3x Long MicroStrategy Daily ETP tries to replicate the performance of the Solactive Daily Leveraged 3x Long MicroStrategy Index, which in turn seeks total return

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